

# A Microstructure Analysis of the Carbon Finance Market

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- EU Emissions Trading Scheme (ETS)
- Price and volume of contracts
- European Pilot Scheme
- Prior literature
- A Microstructure perspective
- Some preliminary results

# EU Emissions Trading Scheme

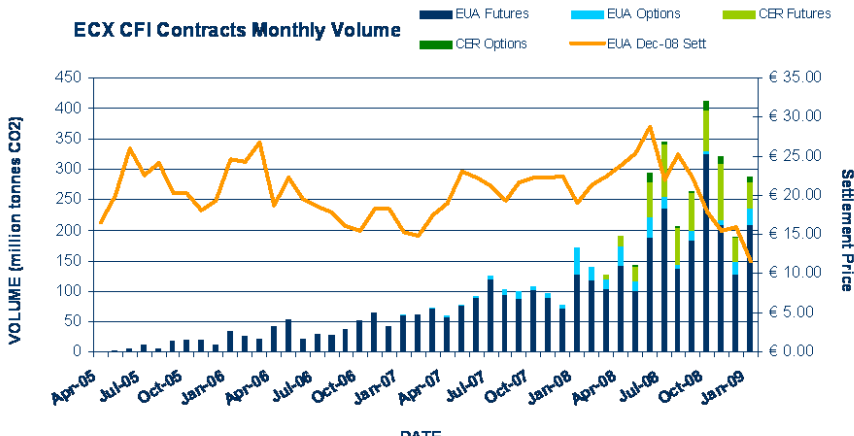
- Largest multi-national, carbon trading program is the European Union Emission Trading Scheme (ETS) to trade European Union Allowances (EUA), which began in 2005 under Kyoto.
  - First compliance period (pilot phase) 2005-2007.
  - Second compliance period (Kyoto phase) 2008-2012.
  - Third phase 2013-2020.
- The ETS covers over 10,000 EU energy and industrial installations. These covered facilities account for an estimated 50% of CO<sub>2</sub> and greenhouse gas EU emissions.
- Covered emitters [participating firms] are allocated allowances [EUAs] by member states without charge according to their National Allocation Plans.

# EU Emissions Trading Scheme

- Firms have to reduce the amount of emitted CO<sub>2</sub> and annually demonstrate that their level of EUAs corresponds to their actual emissions.
  - Each February member states allocate the EUAs to compliant firms.
  - April 30 the following year, firms must deliver the required EUAs to the national surveillance authorities in accordance with their actual emissions volume. Non-compliance results in additional fines.
- Firms able to keep emissions below their allocation can sell excess EUAs in the market.
- Firms which require additional allowances must either invest in emissions-reducing technologies or buy EUAs in the market.

# Price and volume ECX CFI Contracts

- 94 million tonnes of CO<sub>2</sub> traded in 2005 with a market value of 2.1 billion euro
- 452 million tonnes of CO<sub>2</sub> traded in 2006 with a market value of 9 billion euro
- 1 billion tonnes of CO<sub>2</sub> traded in 2007 with a market value of 17.5 billion euro
- 2 billion tonnes of CO<sub>2</sub> traded in 2008 (first 9 months) with a market value of 69 billion euro



# European Pilot Phase 2005-07

- Pilot Phase 2005-07: Can a market be set up? **YES**
- Only includes the power sector and heavy industry: cement, steel, oil refining, glass and ceramics, pulp and paper.
- Allowances allocated for free by Member States.
- You could borrow from future years, and bank forward, but only within the 3 year period. i.e. not between phases.
- Power sectors were left short (fewer allowances) the rest were long (more than they needed).
- Most countries over allocated: only UK, Spain, Italy, Ireland and Austria were short, the rest long.
- Linking directive allowed the use of reductions purchased from projects in developing countries certified by the UN (certified emission reductions via the Clean Development Mechanism).

## Spot and Futures Price Dynamics

- Prior studies typically analyse spot market and often only at a daily frequency.
- Daskalakis et al. (2009), Benz and Trück (2009), Seifert et al. (2008), Paoletta and Taschini (2008)

## Economic Models

- A few papers attempt to incorporate economic variables to explain behavior:
- Redmond and Convery (2006), Bredin and Muckley (2009)

## Microstructure

- Benz and Hengelbrock (2009) investigate the ETS from a microstructure perspective but focus on price discovery between spot and futures.

# Investigate from a Micro Structure Perspective

- Investigate the interaction between trade duration, trade volume and price volatility in an attempt to understand information flow.
- Large amount prior research examines focuses on one or two trade processes in isolation, rather than on their combined effects.
- Follow Bowe et al. (2009) in applying Manganeli (2005), Xu et al. (2006) to examine the the relationship between the three processes and how information is revealed.
- Examine these issues in the context of Phase 1 EU ETS futures.

- Price volatility and volume are positively correlated.
  - Mixture of distributions hypothesis (MDH) [Clark, 1973; Epps and Epps, 1976; Tauchen and Pitts, 1983; Harris, 1986; 1987]
  - Sequential information arrival hypothesis [Copeland, 1976; Jennings et al., 1981]
  - Karpoff (1988): trading restrictions, cost asymmetries in equity markets [e.g. higher costs of short selling]. Reports negative correlation for financial futures contracts.
  - Driven by informed trading. Li and Wu (2006) account for informed trading and uncover a negative correlation.
  - Bowe et al. (2009) report negative correlations for interest rate futures.
- Xu et al (2006) use a bivariate VAR to examine the volume-volatility relationship accounting for durations. Shorter durations imply higher probability of news arriving and greater price volatility.
- Manganello (2005) models duration, volume and returns simultaneously. He finds volume clusters, Times of greater activity coincide with higher numbers of informed traders.

## Xu et al. (2006)

- Estimate VAR model between time-standardized volume and volatility.
- The variables are standardized with respect to duration and are in log-form to ensure that they remain positive. The volatility and the volume equations are given as:

$$z_{d,t} = \sum_{i=1}^p a_{z,i} z_{d,t-i} + \sum_{i=0}^q (b_{z,i} + c_{z,i} \tau_{t-i}) v_{d,t-i} + u_t$$
$$v_{d,t} = \sum_{i=1}^p a_{v,i} z_{d,t-i} + \sum_{i=0}^q (b_{v,i} + c_{v,i} \tau_{t-i}) v_{d,t-i} + \varepsilon_t$$

- $z_{d,t}$  and  $v_{d,t}$  are logs of volatility ( $Z_t$ ) and volume ( $V_t$ ) standardized by duration ( $d_t$ ).
- $\tau_t$  is log duration.

- Estimate an unrestricted VAR using duration, volume and volatility.

$$\begin{aligned}d_t &= \sum_{i=1}^{p1} \gamma_i d_{t-i} + \sum_{i=1}^{q1} \rho_i V_{t-i} + \sum_{i=1}^{r1} \delta_i Z_{t-i} + \varepsilon_t \\V_t &= \sum_{i=0}^{p2} \lambda_i d_{t-i} + \sum_{i=0}^{q2} \zeta_i V_{t-i} + \sum_{i=0}^{r2} \phi_i Z_{t-i} + \eta_t \\Z_t &= \sum_{i=0}^{p3} \beta_i d_{t-i} + \sum_{i=0}^{q3} \theta_i V_{t-i} + \sum_{i=1}^{r3} \alpha_i Z_{t-i} + u_t\end{aligned}$$

- $d_t$  is log duration,  $V_t$  is log volume and  $Z_t$  is log volatility.

- Sample: Phase 1 EU ETS Dec. 05, 06, 07 and 08 Futures
- 1 futures contract corresponds to 1,000 EUAs and represents the right to emit 1,000 tonnes of CO<sub>2</sub>.
  - December 2005: April 22, 2005 - December 19, 2005
  - December 2006: June 16, 2005 - December 18, 2006
  - December 2007: June 3, 2005 - May 18, 2007
  - December 2008: September 28, 2005 - May 18, 2007
- Contracts expire on last Monday in December.

- Sample: Phase 1 EU ETS Dec. 05, 06, 07 and 08 Futures
- Data are cleaned: volume of trades recorded at the same time and at the same price are combined, overnight period is eliminated.
- Also adjust for diurnal seasonality in duration and volume.

**Table: Summary Statistics for EU ETS Futures**

| Instrument     | No. Obs | Duration |         | Volume |        |
|----------------|---------|----------|---------|--------|--------|
|                |         | Mean     | Median  | Mean   | Median |
| <b>Dec. 05</b> | 3135    | 1297.836 | 421.761 | 10.318 | 10     |
| <b>Dec. 06</b> | 15831   | 599.274  | 191.000 | 12.072 | 10     |
| <b>Dec. 07</b> | 5118    | 1388.770 | 392.000 | 13.216 | 10     |
| <b>Dec. 08</b> | 5484    | 750.158  | 164.000 | 11.538 | 10     |

## Structural VAR - Duration based variables (Dec 06)

- Evidence that both volatility and volume are persistent.
- Contemporaneous coefficient between volume and volatility is negative and significant – contrary to the MDH.
- Coefficient on Duration\*volume also negative and highly significant.

## Structural VAR (Dec. 2006)

- Duration is persistent. It current duration positively impacts both volume and volatility. Lagged duration has a negative impact.
- Volume positively impacts duration, no significant impact from (current) volatility.
- Evidence that both volatility and volume are persistent.
- Contemporaneous coefficient between volume and volatility is negative and significant – contrary to the MDH.

- Evidence that the market has been liquid, and trading volume has grown significantly.
- There was some modest abatement.
- Negative relationship between volume and volatility contrary to MDH but consistent with prior findings in futures markets.
- Evidence to suggest duration analysis is important consideration.

# Structural VAR - Duration based variables (Dec 06)

|                     | Volatility Equation |             |         | Volume Equation |         |
|---------------------|---------------------|-------------|---------|-----------------|---------|
|                     |                     | Coefficient | t-stats | Coefficient     | t-stats |
| Lagged volatility   | $a_1$               | 0.168       | 21.121  | -0.002          | -1.511  |
|                     | $a_2$               | 0.057       | 7.071   |                 |         |
|                     | $a_3$               | 0.061       | 7.563   |                 |         |
|                     | $a_4$               | 0.063       | 7.719   |                 |         |
|                     | $a_5$               | 0.020       | 2.477   |                 |         |
|                     | $a_6$               | 0.060       | 7.335   |                 |         |
|                     | $a_7$               | 0.022       | 2.796   |                 |         |
|                     | $a_8$               | 0.053       | 6.547   |                 |         |
|                     | $a_9$               | 0.017       | 2.073   |                 |         |
| Current volume      | $b_0$               | -0.304      | -7.788  |                 |         |
| Lagged volume       | $b_1$               |             |         | 0.174           | 21.765  |
|                     | $b_2$               |             |         | 0.088           | 10.814  |
|                     | $b_3$               |             |         | 0.053           | 6.503   |
|                     | $b_4$               | 0.099       | 2.470   | 0.030           | 3.703   |
|                     | $b_5$               |             |         | 0.032           | 3.938   |
|                     | $b_6$               |             |         | 0.029           | 3.511   |
|                     | $b_7$               |             |         | 0.043           | 5.306   |
|                     | $b_8$               |             |         | 0.031           | 3.760   |
|                     | $b_9$               |             |         | 0.019           | 2.315   |
| Current dur.*volume | $c_0$               | -0.233      | -6.470  |                 |         |
| Lagged dur.*volume  | $c_1$               |             |         | -0.040          | -5.473  |

# Structural VAR (Dec. 2006)

|                    | Duration Equation |             |         | Volume Equation |         | Volatility Equation |         |
|--------------------|-------------------|-------------|---------|-----------------|---------|---------------------|---------|
|                    |                   | Coefficient | t-stats | Coefficient     | t-stats | Coefficient         | t-stats |
| Current duration   | $\gamma_0$        |             |         | 0.050           | 6.515   | 1.145               | 21.567  |
| Lagged duration    | $\gamma_1$        | 0.220       | 27.217  |                 |         | -0.208              | -3.769  |
|                    | $\gamma_2$        | 0.095       | 11.484  |                 |         |                     |         |
|                    | $\gamma_3$        | 0.053       | 6.328   |                 |         |                     |         |
|                    | $\gamma_4$        | 0.019       | 2.272   | -0.027          | -3.442  | -0.233              | -4.203  |
|                    | $\gamma_5$        | 0.031       | 3.718   |                 |         |                     |         |
|                    | $\gamma_6$        | 0.020       | 2.434   |                 |         |                     |         |
|                    | $\gamma_7$        | 0.032       | 3.829   |                 |         |                     |         |
|                    | $\gamma_8$        | 0.020       | 2.445   |                 |         |                     |         |
|                    | $\gamma_9$        | 0.018       | 2.134   |                 |         |                     |         |
| Current volume     | $\rho_0$          | 0.046       | 5.533   |                 |         | -0.487              | -8.808  |
| Lagged volume      | $\rho_1$          |             |         | 0.175           | 21.989  | 0.122               | 2.166   |
|                    | $\rho_2$          |             |         | 0.077           | 9.485   |                     |         |
|                    | $\rho_3$          |             |         | 0.059           | 7.235   |                     |         |
|                    | $\rho_4$          |             |         | 0.033           | 4.048   |                     |         |
|                    | $\rho_5$          |             |         | 0.031           | 3.864   |                     |         |
|                    | $\rho_6$          | -0.018      | -2.071  | 0.023           | 2.830   |                     |         |
|                    | $\rho_7$          |             |         | 0.037           | 4.601   |                     |         |
|                    | $\rho_8$          |             |         | 0.027           | 3.357   |                     |         |
| Current volatility | $\delta_0$        |             |         | -0.010          | -8.808  |                     |         |
| Lagged volatility  | $\delta_1$        |             |         |                 |         | 0.201               | 11.197  |
|                    | $\delta_2$        | -0.004      | -3.163  |                 |         | 0.098               | 5.341   |
|                    | $\delta_3$        | -0.003      | -2.289  |                 |         | 0.065               | 3.561   |
|                    | $\delta_4$        |             |         | 0.003           | 2.289   |                     |         |
|                    | $\delta_5$        | -0.003      | -2.696  |                 |         | 0.129               | 7.062   |
|                    | $\delta_6$        |             |         |                 |         |                     |         |
|                    | $\delta_7$        | -0.003      | -2.151  |                 |         |                     |         |
|                    | $\delta_8$        |             |         |                 |         |                     |         |
|                    | $\delta_9$        |             |         |                 |         |                     |         |

# Autoregressive coefficients ( $\beta$ ) (Dec. 2005-2008 Contracts)

This table reports the Autoregressive coefficients ( $\beta$ ) for Duration, Volume and Trade Variance models:

$$\begin{aligned}d_t &= \psi_t \epsilon_t, \quad \epsilon_t \sim i.i.d.(1, \sigma_\epsilon^2) \\ \psi_t &= \omega + \alpha d_{t-1} + \beta \psi_{t-1}\end{aligned}\tag{1}$$

$$\begin{aligned}v_t &= \phi_t \eta_t, \quad \eta_t \sim i.i.d.(1, \sigma_\eta^2) \\ \phi_t &= \omega + \alpha v_{t-1} + \beta \phi_{t-1}\end{aligned}\tag{2}$$

$$\begin{aligned}y_t &= \sigma_t \zeta_t, \quad \zeta_t \sim i.i.d.(0, 1) \\ \sigma_t^2 &= \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2\end{aligned}\tag{3}$$

Estimation is based on the tick-by-tick data for the December 05-08 EU ETS futures contract on the ECX.

| EU ETS futures contract | ACD               | ACV               | GARCH              |
|-------------------------|-------------------|-------------------|--------------------|
| December 2005           | 0.644<br>(24.258) | 0.803<br>(11.025) | 0.950<br>(192.602) |
| December 2006           | 0.696<br>(15.039) | 0.750<br>(16.266) | 0.968<br>(594.44)  |
| December 2007           | 0.559<br>(6.976)  | 0.481<br>(5.338)  | 0.929<br>(244.370) |
| December 2008           | 0.397<br>(6.221)  | 0.709<br>(13.074) | 0.962<br>(299.223) |

# Structural VAR - Duration based variables (Dec 05)

$$z_{d,t} = \sum_{i=1}^p a_{zi} z_{d,t-i} + \sum_{i=0}^q (b_{zi} + c_{zi} \tau_{t-i}) v_{d,t-i} + u_t \quad (4)$$

$$v_{d,t} = \sum_{i=1}^p a_{vi} z_{d,t-i} + \sum_{i=1}^q (b_{vi} + c_{vi} \tau_{t-i}) v_{d,t-i} + \epsilon_t$$

|                     | Volatility Equation |             |         | Volume Equation |         |
|---------------------|---------------------|-------------|---------|-----------------|---------|
|                     |                     | Coefficient | t-stats | Coefficient     | t-stats |
| Lagged volatility   | $a_1$               | 0.185       | 10.307  | -0.017          | -2.899  |
|                     | $a_2$               | 0.091       | 4.975   | 0.007           | 1.157   |
|                     | $a_3$               | 0.056       | 3.066   | 0.006           | 0.981   |
|                     | $a_5$               | 0.117       | 8.391   | 0.003           | 0.474   |
| Current volume      | $b_0$               | -0.300      | -5.257  |                 |         |
| Lagged volume       | $b_1$               | 0.077       | 1.309   | 0.207           | 11.346  |
|                     | $b_2$               | 0.009       | 0.156   | 0.084           | 4.485   |
|                     | $b_3$               | -0.009      | -0.163  | 0.091           | 4.857   |
|                     | $b_5$               | 0.041       | 0.713   | 0.061           | 3.233   |
|                     | $b_7$               | -0.124      | -2.102  | 0.002           | 0.117   |
|                     | $b_9$               | -0.009      | -0.148  | 0.061           | 3.242   |
| Current dur.*volume | $c_0$               | -0.233      | -6.470  |                 |         |
| Lagged dur.*volume  | $c_1$               | 0.049       | 1.340   | -0.040          | -5.473  |

# Structural VAR - Duration based variables (Dec. 07)

|                     | Volatility Equation |             |         | Volume Equation |         |
|---------------------|---------------------|-------------|---------|-----------------|---------|
|                     |                     | Coefficient | t-stats | Coefficient     | t-stats |
| Lagged volatility   | $a_1$               | 0.349       | 25.517  | -0.006          | -2.100  |
|                     | $a_2$               | -0.068      | -4.650  | 0.006           | 2.040   |
|                     | $a_3$               | 0.051       | 3.455   | -0.007          | -2.343  |
|                     | $a_4$               | -0.023      | -1.548  | 0.008           | 2.748   |
|                     | $a_5$               | 0.037       | 2.540   | 0.007           | 2.388   |
|                     | $a_6$               | 0.034       | 2.283   | 0.001           | 0.312   |
|                     | $a_8$               | 0.039       | 2.624   | 0.002           | 0.627   |
|                     | $a_{10}$            | 0.033       | 2.255   | -0.004          | -1.259  |
|                     | $a_{13}$            | 0.056       | 3.829   | -0.004          | -1.453  |
|                     | $a_{14}$            | -0.033      | -2.272  | 0.004           | 1.394   |
|                     | $a_{15}$            | 0.034       | 2.328   | -0.001          | -0.506  |
|                     | $a_{18}$            | 0.038       | 2.583   | 0.000           | 0.054   |
| Current volume      | $b_0$               | -0.293      | -4.233  |                 |         |
| Lagged volume       | $b_1$               | 0.094       | 1.324   | 0.222           | 15.705  |
|                     | $b_2$               | -0.026      | -0.368  | 0.086           | 5.947   |
|                     | $b_3$               | 0.104       | 1.460   | 0.049           | 3.372   |
|                     | $b_5$               | 0.149       | 2.089   | 0.043           | 2.930   |
|                     | $b_9$               | 0.154       | 2.154   | 0.007           | 0.472   |
|                     | $b_{11}$            | -0.083      | -1.162  | 0.035           | 2.404   |
| Current dur.*volume | $c_0$               | -0.095      | -1.528  |                 |         |
| Lagged dur.*volume  | $c_1$               | -0.143      | -2.236  | -0.054          | -4.245  |
|                     | $c_4$               | -0.163      | -2.546  | 0.031           | 2.358   |
|                     | $c_5$               | 0.143       | 2.233   | -0.006          | -0.471  |
|                     | $c_9$               | -0.191      | -2.966  | -0.033          | -2.534  |

# Structural VAR - Duration based variables (Dec. 08)

$$z_{d,t} = \sum_{i=1}^p a_{zi} z_{d,t-i} + \sum_{i=0}^q (b_{zi} + c_{zi} \tau_{t-i}) v_{d,t-i} + u_t \quad (5)$$

$$v_{d,t} = \sum_{i=1}^p a_{vi} z_{d,t-i} + \sum_{i=1}^q (b_{vi} + c_{vi} \tau_{t-i}) v_{d,t-i} + \varepsilon_t$$

|                     | Volatility Equation |             |         | Volume Equation |         |        |
|---------------------|---------------------|-------------|---------|-----------------|---------|--------|
|                     |                     | Coefficient | t-stats | Coefficient     | t-stats |        |
| Lagged volatility   | $a_1$               | 0.125       | 9.248   | -0.011          | -2.427  |        |
|                     | $a_2$               | 0.059       | 4.290   | -0.001          | -0.147  |        |
|                     | $a_3$               | 0.064       | 4.712   | -0.003          | -0.752  |        |
|                     | $a_4$               | -0.032      | -2.326  | 0.002           | 0.374   |        |
|                     | $a_5$               | 0.033       | 2.446   | -0.003          | -0.642  |        |
|                     | $a_7$               | 0.039       | 2.858   | 0.005           | 1.063   |        |
|                     | $a_9$               | 0.037       | 2.731   | 0.005           | 1.071   |        |
|                     | $a_{10}$            | 0.037       | 2.724   | 0.011           | 2.486   |        |
|                     | Current volume      | $b_0$       | -0.326  | -7.856          |         |        |
|                     | Lagged volume       | $b_1$       | 0.033   | 0.779           | 0.233   | 17.122 |
| $b_2$               |                     | -0.038      | -0.892  | 0.107           | 7.628   |        |
| $b_3$               |                     | -0.067      | -1.555  | 0.078           | 5.500   |        |
| Current dur.*volume | $c_0$               | -0.016      | -4.231  |                 |         |        |
| Lagged dur.*volume  | $c_1$               | -0.043      | -1.106  | -0.075          | -6.095  |        |
|                     | $c_3$               | 0.013       | 0.331   | 0.026           | 2.029   |        |

Note:

# Structural VAR (Dec. 2005)

|                    |            | Duration Equation |         | Volume Equation |         | Volatility Equation |         |
|--------------------|------------|-------------------|---------|-----------------|---------|---------------------|---------|
|                    |            | Coefficient       | t-stats | Coefficient     | t-stats | Coefficient         | t-stats |
| Current duration   | $\gamma_0$ |                   |         |                 |         | 1.335               | 15.361  |
| Lagged duration    | $\gamma_1$ | 0.218             | 11.729  |                 |         | -0.362              | -3.361  |
|                    | $\gamma_2$ | 0.096             | 5.030   |                 |         |                     |         |
|                    | $\gamma_3$ | 0.058             | 3.059   |                 |         | -0.238              | -2.565  |
|                    | $\gamma_4$ | 0.048             | 2.498   |                 |         |                     |         |
|                    | $\gamma_5$ | 0.065             | 3.393   |                 |         | -0.268              | -2.882  |
|                    | $\gamma_7$ |                   |         | 0.046           | 2.141   |                     |         |
| Current volume     | $\rho_0$   |                   |         |                 |         | -0.277              | -3.681  |
| Lagged volume      | $\rho_1$   |                   |         | 0.178           | 9.917   |                     |         |
|                    | $\rho_2$   |                   |         | 0.040           | 2.185   |                     |         |
|                    | $\rho_3$   |                   |         | 0.052           | 2.879   | -0.206              | -2.702  |
|                    | $\rho_5$   |                   |         | 0.054           | 2.989   |                     |         |
|                    | $\rho_7$   |                   |         | 0.036           | 2.030   | -0.165              | -2.192  |
| Current volatility | $\delta_0$ |                   |         | -0.016          | -3.681  |                     |         |
| Lagged volatility  | $\delta_1$ |                   |         | -0.011          | -2.550  | 0.201               | 11.197  |
|                    | $\delta_2$ |                   |         |                 |         | 0.098               | 5.341   |
|                    | $\delta_3$ |                   |         |                 |         | 0.065               | 3.561   |
|                    | $\delta_5$ |                   |         |                 |         | 0.129               | 7.062   |
|                    | $\delta_7$ |                   |         | -0.010          | -2.265  |                     |         |

# Structural VAR (Dec. 2007)

|                    |            | Duration Equation |         | Volume Equation |         | Volatility Equation |         |
|--------------------|------------|-------------------|---------|-----------------|---------|---------------------|---------|
|                    |            | Coefficient       | t-stats | Coefficient     | t-stats | Coefficient         | t-stats |
| Current duration   | $\gamma_0$ |                   |         | 0.109           | 7.669   | 1.252               | 13.459  |
| Lagged duration    | $\gamma_1$ | 0.275             | 19.090  |                 |         | -0.562              | -5.748  |
|                    | $\gamma_2$ | 0.901             | 6.076   |                 |         |                     |         |
|                    | $\gamma_3$ | 0.064             | 4.250   |                 |         |                     |         |
|                    | $\gamma_5$ | 0.036             | 2.414   |                 |         |                     |         |
|                    | $\gamma_9$ |                   |         |                 |         | -0.364              | -3.689  |
| Current volume     | $\rho_0$   | 0.043             | 3.061   |                 |         | -0.336              | -3.605  |
| Lagged volume      | $\rho_1$   |                   |         | 0.223           | 15.798  |                     |         |
|                    | $\rho_2$   |                   |         | 0.087           | 6.019   |                     |         |
|                    | $\rho_3$   |                   |         | 0.040           | 2.780   | 0.214               | 2.230   |
|                    | $\rho_4$   |                   |         | 0.030           | 2.030   | -0.227              | -2.360  |
|                    | $\rho_5$   |                   |         |                 |         | 0.320               | 3.330   |
|                    | $\rho_6$   |                   |         | 0.030           | 2.000   |                     |         |
|                    | $\rho_9$   |                   |         | -0.032          | -2.226  |                     |         |
| Current volatility | $\delta_0$ |                   |         | -0.008          | -3.605  |                     |         |
| Lagged volatility  | $\delta_1$ |                   |         |                 |         | 0.350               | 25.590  |
|                    | $\delta_2$ | -0.006            | -2.538  |                 |         | -0.069              | -4.711  |
|                    | $\delta_3$ |                   |         | -0.007          | -3.269  | 0.051               | 3.488   |
|                    | $\delta_4$ |                   |         | 0.005           | 2.102   |                     |         |
|                    | $\delta_5$ |                   |         | 0.005           | 2.177   | 0.036               | 2.439   |
|                    | $\delta_6$ |                   |         |                 |         | 0.035               | 2.399   |
|                    | $\delta_8$ |                   |         |                 |         | 0.038               | 2.611   |

# Structural VAR (Dec. 2008)

|                    | Duration Equation |             |         | Volume Equation |         | Volatility Equation |         |
|--------------------|-------------------|-------------|---------|-----------------|---------|---------------------|---------|
|                    |                   | Coefficient | t-stats | Coefficient     | t-stats | Coefficient         | t-stats |
| Current duration   | $\gamma_0$        |             |         | 0.069           | 5.099   | 1.189               | 21.015  |
| Lagged duration    | $\gamma_1$        | 0.314       | 22.246  |                 |         | -0.221              | -3.604  |
|                    | $\gamma_2$        | 0.094       | 6.402   |                 |         |                     |         |
|                    | $\gamma_3$        |             |         |                 |         |                     |         |
|                    | $\gamma_5$        | 0.042       | 2.863   |                 |         |                     |         |
|                    | $\gamma_7$        | 0.033       | 2.787   |                 |         |                     |         |
|                    | $\gamma_9$        | 0.033       | 2.229   |                 |         |                     |         |
| Current volume     | $\rho_0$          |             |         |                 |         | -0.467              | -8.001  |
| Lagged volume      | $\rho_1$          |             |         | 0.183           | 13.523  |                     |         |
|                    | $\rho_2$          |             |         | 0.120           | 8.757   |                     |         |
|                    | $\rho_3$          |             |         | 0.111           | 7.987   |                     |         |
|                    | $\rho_6$          |             |         | 0.037           | 2.674   |                     |         |
|                    | $\rho_7$          |             |         | 0.029           | 2.049   |                     |         |
|                    | $\rho_9$          | -0.029      | -2.047  |                 |         |                     |         |
| Current volatility | $\delta_0$        |             |         | -0.025          | -8.001  |                     |         |
| Lagged volatility  | $\delta_1$        |             |         |                 |         | 0.126               | 9.300   |
|                    | $\delta_2$        |             |         |                 |         | 0.059               | 4.329   |
|                    | $\delta_3$        |             |         |                 |         | 0.065               | 4.734   |
|                    | $\delta_4$        |             |         |                 |         | -0.031              | -2.258  |
|                    | $\delta_5$        |             |         |                 |         | 0.034               | 2.502   |
|                    | $\delta_7$        |             |         |                 |         | 0.039               | 2.870   |
|                    | $\delta_9$        |             |         |                 |         | 0.038               | 2.786   |

Note: