

**THE VALUE OF TIME DIVERSIFICATION IN THE
MARKOWITZ OPTIMAL PORTFOLIO DECISION
WITH AUSTRALIAN AND INTERNATIONAL ASSET CLASSES**

Lakshman Alles

School of Economics and Finance
Curtin University of Technology
GPO Box U1987
Perth 6001, Australia
Tel. (61) 89266 7811 Fax. (61) 89266 3026
Email: Lakshman.Alles@cbs.curtin.edu.au

and

Bhavesh Haria

Alinta Limited
Level 5, The Quadrant
1 William Street, Perth WA 6000
Phone (08) 9486 3191 (w) or 0422 610 936 (m)
Email: bhavesh.haria@alinta.net.au

Objective of the paper

- To evaluate the relative roles played by diversification through time compared to diversification through asset classes in achieving optimal investment outcomes, from a range of different investor perspectives.

Motivation for this investigation:

- Divergence in viewpoints among academics and practitioners regarding the implications of making investments over different time horizons - "the time diversification debate."
- Conventional wisdom among practitioners is that it is appropriate to invest more in equities when investment horizon is long but invest more in safer asset classes when the horizon is short.
- The empirical evidence on the aspect of long horizon versus short horizon risk and return analysis is scarce due to data limitations.
- Re-sampling techniques adopted to extend the span of actual datasets to examine long horizon results in US based studies (e.g. Butler and Domian (1991) and Hickman et. al. (2001).
- The optimal asset allocation decision in a multi-asset-class context from an Australian investor viewpoint so far not addressed in the literature.

Issues of practical importance to investors addressed in the study

- Examines the time diversification vs. asset class diversification issue from the Australian investors viewpoint
- Examine the optimal mix of assets in a multi asset class context through the Markowitz portfolio optimization framework
- The optimal mix of assets is examined from a range of different investor viewpoints, with different risk profile assumptions
- Examines the changes to the optimal asset mix as investor horizon changes from the short term to the long term

Investment time horizons examined:

- One year
- Three years
- Five years
- Ten years
- Fifteen years

Assets classes examined:

- Large cap Australian stocks
- Small-cap Australian stocks
- International developed markets stocks
- International emerging markets stocks
- Australian corporate bonds and
- Australian Government bonds

Investor profiles and investment styles assumed:

Retail investor (Conservative) –

- Domestic asset classes only
- Large stocks, corporate bonds and government bonds.
- No short-selling strategies

Retail investor (Aggressive) –

- Domestic asset classes only
- Large stocks, small stocks and corporate bonds
- No short-selling strategies

Local fund manager (Balanced) –

- Caters to conservative institutional investors (Super funds)
- Domestic and developed international asset markets only
- Large stocks, small stocks, government bonds and corporate bonds.
- Follow a strategic asset allocation strategy

Local Fund Manager (Growth) -

- Caters to aggressive investors wanting exposure to international markets.
- Large stocks, small stocks, International developed markets and Australian corporate bonds.

Local Hedge fund –

- Aggressive investment profile
- Large stocks, small stocks, international developed markets and emerging markets.

International Hedge Fund -

- Large stocks, international developed markets, international emerging markets and Australian corporate bonds.

Methodology

To determine optimal portfolio weights, we solve:

$$\begin{bmatrix} r_1 \\ \cdot \\ \cdot \\ \cdot \\ r_n \end{bmatrix} - \begin{bmatrix} R_f \\ \cdot \\ \cdot \\ \cdot \\ R_f \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \cdot & \cdot & \cdot & \sigma_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \sigma_{n1} & \cdot & \cdot & \cdot & \sigma_{nn} \end{bmatrix} \begin{bmatrix} z_1 \\ \cdot \\ \cdot \\ \cdot \\ z_n \end{bmatrix}$$

$$\mathbf{R} - \mathbf{C} = \mathbf{S} \cdot \mathbf{Z}$$

Where \mathbf{R} is the vector of returns, \mathbf{C} is the risk free rate and \mathbf{S} is the variance covariance matrix

$$\therefore \mathbf{Z} = \mathbf{S}^{-1} [\mathbf{R} - \mathbf{C}]$$

Optimal portfolio weights $\mathbf{W}^T = [w_1, w_2, \dots, w_n]$ are calculated as

$$w_i = \frac{z_i}{\sum z_i}$$

Return of the optimal portfolio return is then calculated as

$$R_p = [\mathbf{W}^T] [\mathbf{R}_i]$$

Variance of the optimal portfolio is given by

$$V = [\mathbf{W}^T] [\mathbf{S}] [\mathbf{W}]$$

And the Sharpe ratio of the optimal portfolio is shown by

$$(R_p - R_f)/V^{1/2}$$

Data, sample period and proxies

The sample period: December 1984 to September 2005

Proxies used for asset classes:

Large stocks: S&P ASX 100 Accumulation index

Small stocks: S&P Small Ordinaries index

International stocks : Morgan Stanley Capital International (MSCI) World ex Australia index

Emerging market stocks: International Financial Corporation Emerging markets Composite index

Corporate Bonds: JP Morgan Total Return Corporate Bond index

Australian Government bonds: 10 year Commonwealth bond index from the Reserve Bank of Australia

Proxy for the risk free rate: 90-day bank accepted bill rate from the RBA

Return Computations:

Stock returns are fully adjusted for distributions.

All return computations are based on continuous compounding.

Holding period returns computation by re-sampling

Randomly re-sample the available data (with replacement) using the bootstrap methodology, to compute required holding period return.

Variants of the restrictions on asset weights assumed in the mean-variance optimisation model:

- No asset weight restrictions
- Short-sale restrictions only
- Asset weight restrictions $0.05 < w_i < 0.5$

Figure 1

Average Monthly Return, Variance and Standard Deviation
January 1985 – September 2005

	Return	Variance	Std Dev
Large Stocks	1.22%	0.23%	4.82%
Small Stocks	1.08%	0.25%	5.03%
Global Developed Markets	1.19%	0.18%	4.25%
Global Emerging Markets	1.18%	0.41%	6.39%
Australian Corporate Bonds	1.02%	0.13%	3.54%
Australian Government Bonds	0.71%	0.00%	0.24%
90-day Bank Bills	0.69%	0.00%	0.34%

Figure 3

Returns and Standard Deviations
Annualised Holding period returns

ANNUALISED RETURNS AND STANDARDIZED DEVIATIONS					
	12 Month	3 Years	5 years	10 years	15 Years
Large Stocks	12.76% (19.03%)	10.74% (10.14%)	9.64% (9.13%)	8.42% (3.10%)	7.26% (1.98%)
Small Stocks	11.21% (19.61%)	9.39% (10.09%)	7.88% (12.26%)	7.47% (3.37%)	6.22% (2.37%)
International Developed	13.36% (15.02%)	11.16% (6.88%)	10.24% (4.65%)	8.63% (2.30%)	7.37% (1.44%)
Emerging Markets	11.16% (22.25%)	9.51% (11.00%)	8.12% (13.57%)	7.21% (4.27%)	6.53% (2.59%)
Aust Corp Bonds	11.54% (12.16%)	10.18% (6.08%)	9.17% (3.88%)	7.77% (2.01%)	6.75% (1.30%)
Aust Govt 10yr Bonds	8.83% (0.96%)	8.15% (0.44%)	7.58% (0.30%)	6.53% (0.17%)	5.78% (0.11%)
Bank Bill	8.20% (1.23%)	7.61% (0.58%)	7.11% (0.40%)	6.17% (0.23%)	5.49% (0.14%)

Figure 4 – Optimal Portfolios without restrictions

ASSET WEIGHTS IN PORTFOLIO

Assets		12 Month	5 year	5 year	10 year	15 year
Conservative Retail Investor Portfolio	Large Stocks	2.1%	2.0%	1.9%	2.0%	1.9%
	Aust Corp Bonds	2.4%	2.0%	1.7%	2.0%	2.0%
	Aust Govt 10yr Bonds	76.5%	76.0%	76.4%	76.2%	76.2%
Aggressive Retail Investor	Large Stocks	69.9%	72.6%	66.0%	77.1%	74.1%
	Small Stocks	-36.0%	-36.7%	-60.2%	-35.7%	-37.4%
	Aust Corp Bonds	64.4%	64.2%	64.8%	61.6%	61.8%
Australien Fund Manager (Balanced)	Large Stocks	1.0%	1.2%	1.1%	1.4%	0.9%
	International Developed	2.4%	1.8%	1.7%	2.0%	2.1%
	Aust Corp Bonds	2.4%	2.2%	1.6%	2.7%	2.4%
	Aust Govt 10yr Bonds	74.2%	74.6%	76.6%	73.9%	74.6%
Australien Fund Manager (Growth)	Large Stocks	39.6%	39.7%	52.7%	46.0%	42.7%
	Small Stocks	-34.1%	-30.7%	-44.6%	-36.9%	-32.0%
	International Developed	40.6%	40.2%	42.6%	40.0%	41.0%
	AUST Corp Bonds	60.9%	60.6%	49.0%	49.8%	48.8%
International Hedge Fund	Large Stocks	12.0%	12.4%	16.0%	17.6%	16.7%
	International Developed	44.9%	37.0%	41.0%	40.6%	40.9%
	Emerging Markets	-4.0%	1.9%	-2.9%	-6.6%	-3.0%
	Aust Corp Bonds	49.1%	48.2%	46.1%	47.8%	46.8%
Australien Hedge Fund	Large Stocks	31.9%	73.1%	54.6%	34.2%	36.2%
	Small Stocks	-67.4%	-49.7%	-74.0%	-65.8%	-69.2%
	International Developed	100.0%	34.9%	93.0%	91.9%	94.6%
	Emerging Markets	-28.0%	-8.8%	-18.1%	-17.0%	-12.6%

Figure 4 – Optimal Portfolios without restrictions

PORTFOLIO RETURN, VOLATILITY AND SHARPE RATIO

		12 Month	8 year	8 year	10 year	15 year
Conservative Retail Investor Portfolio	Assets					
	Portfolio Return	9.0%	24.9%	44.7%	89.9%	154.2%
	Standard Deviation	1.1%	1.7%	2.2%	3.3%	3.9%
	Sharpe's Ratio	0.74	1.33	1.69	2.37	2.93
Aggressive Retail Investor	Portfolio Return	12.6%	33.1%	64.6%	120.2%	189.6%
	Standard Deviation	10.6%	10.6%	26.0%	33.7%	42.4%
	Sharpe's Ratio	0.41	0.73	0.91	1.37	1.67
Australian Fund Manager (Balanced)	Portfolio Return	9.0%	27.0%	44.9%	90.6%	156.0%
	Standard Deviation	1.1%	1.8%	2.3%	3.6%	4.0%
	Sharpe's Ratio	0.77	1.37	1.73	2.48	3.03
Australian Fund Manager (Growth)	Portfolio Return	12.9%	33.4%	65.6%	130.4%	191.9%
	Standard Deviation	9.8%	16.3%	23.6%	30.1%	37.3%
	Sharpe's Ratio	0.43	0.84	1.04	1.61	1.86
International Hedge Fund	Portfolio Return	12.6%	36.9%	61.7%	126.3%	184.7%
	Standard Deviation	9.4%	16.4%	21.7%	28.7%	36.4%
	Sharpe's Ratio	0.46	0.80	0.96	1.61	1.76
Australian Hedge Fund	Portfolio Return	14.8%	41.6%	73.6%	149.0%	213.6%
	Standard Deviation	17.1%	26.7%	37.3%	52.3%	61.6%
	Sharpe's Ratio	0.39	0.64	0.87	1.20	1.47

Figure 5 – Optimal Portfolios with short sale restrictions

ASSET WEIGHTS IN PORTFOLIO

Assets		12 Month	3 year	5 year	10 year	15 year
Conservative Retail Investor Portfolio	Large Stocks	2.1%	2.0%	1.9%	2.3%	1.9%
	Aust Corp Bonds	2.4%	2.0%	1.7%	2.5%	2.0%
	Aust Govt 10yr Bonds	95.5%	96.0%	96.4%	95.2%	96.2%
Aggressive Retail Investor	Large Stocks	36.4%	39.6%	39.2%	41.4%	41.7%
	Small Stocks	0.0%	0.0%	0.0%	0.0%	0.0%
	Aust Corp Bonds	63.6%	60.4%	60.8%	58.6%	58.3%
Australian Fund Manager (Balanced)	Large Stocks	1.0%	1.2%	1.1%	1.4%	0.9%
	International Developed	2.4%	1.8%	1.7%	2.0%	2.1%
	Aust Corp Bonds	2.4%	2.2%	1.6%	2.7%	2.1%
	Aust Govt 10yr Bonds	94.2%	94.8%	95.5%	93.9%	94.8%
Australian Fund Manager (Growth)	Large Stocks	10.0%	13.1%	14.0%	15.2%	14.4%
	Small Stocks	0.0%	0.0%	0.0%	0.0%	0.0%
	International Developed	40.6%	38.8%	39.5%	37.4%	39.1%
	Aust Corp Bonds	49.4%	48.0%	46.5%	47.5%	46.5%
International Hedge Fund	Large Stocks	10.0%	12.4%	14.0%	15.2%	14.4%
	International Developed	40.6%	37.5%	39.5%	37.4%	39.1%
	Emerging Markets	0.0%	1.9%	0.0%	0.0%	0.0%
	Aust Corp Bonds	49.4%	48.2%	46.5%	47.5%	46.5%
Australian Hedge Fund	Large Stocks	13.9%	25.8%	22.9%	27.2%	27.6%
	Small Stocks	0.0%	0.0%	0.0%	0.0%	0.0%
	International Developed	86.1%	74.2%	77.1%	72.8%	72.4%
	Emerging Markets	0.0%	0.0%	0.0%	0.0%	0.0%

Figure 5 – Optimal Portfolios with short sale restrictions

		12 Month	5 year	5 year	10 year	15 year
Assets						
Conservative Retail Investor Portfolio	Portfolio Return	9.0%	26.9%	44.7%	69.9%	104.2%
	Standard Deviation	1.1%	1.7%	2.2%	3.0%	3.9%
	Sharpe's Ratio	0.74	1.55	1.69	2.37	2.93
Aggressive Retail Investor	Portfolio Return	12.0%	36.6%	64.6%	121.7%	180.9%
	Standard Deviation	9.9%	17.4%	26.0%	30.9%	39.1%
	Sharpe's Ratio	0.88	0.60	0.91	1.20	1.40
Australian Fund Manager (Balanced)	Portfolio Return	9.0%	27.0%	44.9%	90.6%	136.0%
	Standard Deviation	1.1%	1.8%	2.3%	3.6%	4.0%
	Sharpe's Ratio	0.77	1.57	1.75	2.43	3.08
Australian Fund Manager (Growth)	Portfolio Return	12.4%	37.0%	61.6%	124.2%	184.0%
	Standard Deviation	9.2%	16.6%	21.6%	25.1%	30.0%
	Sharpe's Ratio	0.46	0.80	0.96	1.60	1.76
International Hedge Fund	Portfolio Return	12.4%	36.9%	61.6%	124.2%	184.0%
	Standard Deviation	9.2%	16.4%	21.6%	25.1%	30.0%
	Sharpe's Ratio	0.46	0.80	0.96	1.60	1.76
Australian Hedge Fund	Portfolio Return	10.3%	41.6%	66.4%	103.0%	196.7%
	Standard Deviation	14.6%	26.7%	31.6%	44.6%	55.7%
	Sharpe's Ratio	0.36	0.64	0.77	1.14	1.36

Figure 6 – Portfolio optimisation with asset weight restrictions of < 5% and < 50%

ASSET WEIGHTS IN PORTFOLIO		12 Month	3 year	5 year	10 year	15 year
Assets						
Conservative Retail Investor Portfolio	Large Stocks	18.1%	19.6%	19.2%	20.3%	20.4%
	Aust Corp Bonds	31.9%	30.4%	30.8%	29.7%	29.6%
	Aust Govt 10yr Bonds	50.0%	50.0%	50.0%	50.0%	50.0%
Aggressive Retail investor	Large Stocks	45.0%	45.0%	45.0%	45.0%	45.0%
	Small Stocks	5.0%	5.0%	5.0%	5.0%	5.0%
	Aust Corp Bonds	50.0%	50.0%	50.0%	50.0%	50.0%
Australian Fund Manager (Balanced)	Large Stocks	5.4%	6.8%	7.2%	7.8%	7.2%
	International Developed	19.6%	19.0%	19.1%	18.1%	19.2%
	Aust Corp Bonds	24.9%	24.2%	23.7%	24.1%	23.6%
	Aust Govt 10yr Bonds	50.0%	50.0%	50.0%	50.0%	50.0%
Australian Fund Manager (Growth)	Large Stocks	5.6%	8.8%	9.6%	10.9%	10.1%
	Small Stocks	5.0%	5.0%	5.0%	5.0%	5.0%
	International Developed	40.5%	38.7%	39.4%	37.1%	38.9%
	Aust Corp Bonds	48.9%	47.4%	45.9%	47.0%	46.0%
International Hedge Fund	Large Stocks	7.9%	11.2%	11.7%	12.9%	12.2%
	International Developed	37.8%	35.5%	36.3%	34.6%	36.3%
	Emerging Markets	5.0%	5.0%	5.0%	5.0%	5.0%
	Aust Corp Bonds	49.4%	48.4%	47.0%	47.4%	46.6%
Australian Hedge Fund	Large Stocks	40.0%	40.0%	40.0%	40.0%	40.0%
	Small Stocks	5.0%	5.0%	5.0%	5.0%	5.0%
	International Developed	50.0%	50.0%	50.0%	50.0%	50.0%
	Emerging Markets	5.0%	5.0%	5.0%	5.0%	5.0%

Figure 6 – Portfolio optimisation with asset weight restrictions of < 5% and < 50%

PORTFOLIO RETURN, VOLATILITY AND SHARPE RATIO

Assets		12 Month	3 year	5 year	10 year	15 year
Conservative Retail Investor Portfolio	Portfolio Return	10.4%	31.5%	52.1%	104.9%	156.5%
	Standard Deviation	5.0%	8.8%	11.7%	15.6%	19.8%
	Sharpe's Ratio	0.44	0.78	0.95	1.46	1.70
Aggressive Retail Investor	Portfolio Return	12.1%	36.7%	60.4%	122.3%	181.6%
	Standard Deviation	10.7%	18.4%	24.6%	32.5%	40.9%
	Sharpe's Ratio	0.38	0.66	0.79	1.24	1.44
Australian Fund Manager (Balanced)	Portfolio Return	10.6%	31.7%	52.8%	106.2%	158.1%
	Standard Deviation	4.7%	7.9%	10.9%	14.3%	17.8%
	Sharpe's Ratio	0.52	0.90	1.08	1.69	1.98
Australian Fund Manager (Growth)	Portfolio Return	12.3%	36.8%	61.0%	123.4%	182.9%
	Standard Deviation	9.2%	15.6%	21.5%	28.3%	35.1%
	Sharpe's Ratio	0.45	0.78	0.93	1.47	1.71
International Hedge Fund	Portfolio Return	12.3%	36.8%	61.1%	123.2%	182.9%
	Standard Deviation	9.1%	15.3%	21.3%	27.9%	34.7%
	Sharpe's Ratio	0.45	0.80	0.95	1.47	1.73
Australian Hedge Fund	Portfolio Return	12.9%	38.5%	64.4%	130.8%	193.2%
	Standard Deviation	14.9%	24.4%	32.5%	45.1%	54.8%
	Sharpe's Ratio	0.52	0.57	0.72	1.08	1.39

Conclusions

- Risk levels decrease significantly as the time horizon increases in all asset classes.
- Portfolio efficiency as measured by the Sharpe ratio increases as the time horizon increases.
- The optimal asset weights do **not** show a great deal of change across different holding periods, indicating that while time-diversification does enhance portfolio efficiency, the relative asset mix in the portfolio is not playing a major part in the efficiency increase.

Limitations of the study

- Some asset class returns did not include returns from dividends. The long-term average was added as an adjustment to the raw returns.
- Implications of the bootstrapping re-sampling procedure used: Time-related information in the data set is lost.
- The mean-variance portfolio optimisation process assumes that the optimisation is a one-period procedure.

Future Research Avenues

- The analysis showed a favorable weighting towards government bonds in the portfolio compositions, possibly due to weak economic conditions making government debt very attractive over the period between 1985 – 1995. An extended dataset reflecting a complete business cycle would provide more balanced results.
- In an extension to this study, it would be useful to add other popular asset classes such as property, commodities, derivatives, hybrids and other classes such as mortgage backed securities, if reliable data for these classes can be obtained.