An Augmented Fama and French Three-Factor Model:

New Evidence From An Emerging Stock Market

Sunil K Bundoo[1]

Department of Economics & Statistics

University of Mauritius

Reduit

Mauritius

Key words: Fama and French Model, Emerging Stock Market

[1] Address correspondence at sbundoo@uom.ac.mu Tel: (230) 7722959

Abstract

There is a lack of empirical evidence of whether the size and value premium are present in emerging equity markets generally, and particularly in the emerging African stock markets. This study provides some empirical evidence in an emerging market, the Stock Exchange of Mauritius, and offers additional out of sample evidence that the size and the book-to-equity effects are international in character. It also innovates by augmenting the Fama and French three-factor model. One may expect that a Fama and French three-factor that takes into account the time-variation in risk, the significance of the size and book-to-market equity effects may be reduced or even disappear. The empirical results confirm that the Fama and French (1993) three factor model holds for the Stock Exchange of Mauritius. Moreover, the empirical results for the augmented model show that the Fama and French three factor model is robust after taking into account time-varying betas.

1.0 Introduction

The Stock Exchange of Mauritius has been in operation for slightly more than 15 years. As at December 2004, there were 40 companies listed on the official market with a market capitalisation of approximately 67 billion rupees and on the Over The Counter (OTC) market there were 78 companies. Trading on the official market started in July 1989, with five listed companies and a market capitalisation of 1.4 billion rupees. Summary statistics on the official market are given in Table 1 below. The trading frequency on the official market has increased progressively from twice weekly to three times weekly in January 1994 before moving to daily trading in November 1997. The stock exchange is run and managed by the Stock Exchange of Mauritius (SEM) Limited and is supervised and regulated by the Financial Services Commission under the Stock Exchange Act 1988. The SEMDEX is the index of all listed ordinary shares and it is a value-weighted index.

Companies listed on the exchange are classified into seven main sectors of the economy, namely, Banks, Insurance and other Finance, Industry, Investments, Sugar, Commerce, Leisure & Hotels and Transport. The rate of corporate tax paid by listed companies is 25 per cent instead of the normal rate of 35 per cent. In the case of tax incentive companies the rate is 15 per cent instead of the normal rate of 25 per cent. There are eleven stock broking companies in operation. A Central Depository and Settlement (CDS) system is operational since January 1997 to speed up share transfer and settlement operations. An electronic trading system (called SEMATS) is operational since June 2001.

TABLE 1 Stock Exchange of Mauritius: Market Highlights

	1989	1991	1994	1997	1999	2001	2003	2004
No. of Listed Companies (Equities)	6	19	34	42	43	40	40	40
Mkt Cap (Rs billion)[1]	1.44	4.86	28.54	36.93	41.73	32.15	51.23	67.03
Mkt Cap ($)[2]	93.26	309.52	1,578.32	1,754.63	1,643.31	1,601.85	1,953.4	2,395.78
Turnover Ratio (%)	0.97	1.67	5.45	8.11	4.74	10.24	5.83	4.21
SEMDEX	117.34	154.17	476.1	391.12	435.69	340.92	549.58	710.77
P/E Ratio	6.56	6.12	20.11	12.86	8.98	5.91	7.43	9.93
Div Yield (%)	5.42	5.11	2.08	3.62	5.03	8.30	5.73	4.84

(Source: SEM Factbooks, various issues)
1 market capitalisation in billion rupees, to 2 d.p.
2 market capitalisation in million US dollars, to 2 d.p.

2.0 Literature Review

Schwert and Seguin (1990) propose and estimate a single factor market model of portfolio returns, which incorporates the estimation of the time-varying component of beta. The Schwert-Seguin (1990) [hereafter SS] model is derived as follows:

The market model is: $$R_{i,t} = \alpha_i + \beta_{i,t} R_{m,t} + e_{i,t} \qquad (1)$$

SS use a heteroscedastic market model showing that betas will vary with the level of aggregate market volatility as follows:

$$\beta_{i,t} = \beta_i + \frac{\delta_i}{\sigma^2_{m,t}} \qquad (2)$$

where β_i is a constant and the time-varying component is given by $\delta_i/\sigma^2_{m,t.}$ β_i is the limit of $\beta_{i,t}$ as the conditional market volatility goes to infinity.

Substitution of (2) into (1) yields the SS market model, as shown below:

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + \delta_i \left(\frac{R_{m,t}}{\sigma^2_{m,t}} \right) + e_{i,t} \qquad (3)$$

where $R_{i,t}$ = return on share i at time t

$R_{m,t}$ = market return at time t

$\sigma^2_{m,t}$ = the conditional market volatility

$e_{i,t}$ = the error term

α, β, δ = are coefficients

The conditional market volatility is estimated using a GARCH (1,1) model. The least squares estimate of δ_i is negative for the small firm portfolio and it is positive for the large firm portfolios.

As market volatility increases, the systematic risk of small firms increases at a faster rate than those of large firms, given that small firms are less diversified and more vulnerable to shocks. Therefore, the spread between the systematic risk of small and large firms is larger during periods of high aggregate market volatility. The Schwert and Seguin results show that the small firm portfolio variances are four times more sensitive to market volatility changes than the large firm portfolio variances.

The Fama and French (1993) three factor asset pricing model was developed as a result of increasing empirical evidence that the Capital Asset Pricing Model performed poorly in explaining realised returns. Fama and French (1993) extended

the Fama and French (1992) study by using a time-series regression approach. The analysis was extended to both stocks and bonds. Monthly returns on stocks and bonds were regressed on five factors: returns on a market portfolio, a portfolio for size and a portfolio for the book-to-market equity effect, a term premium and a default premium. For stocks, the first three factors were found to be significant and for bonds, the last two factors. As a result, Fama and French (FF) construct a three-factor asset pricing model for stocks that includes the conventional market factor and two additional risk factors related to size and book to market equity. They find that this expanded model captures much of the cross section of average returns amongst US stocks.

The model says that the expected return on a portfolio in excess of the risk free rate is explained by the sensitivity of its return to three factors: (i) the excess return on a broad market portfolio, (ii) the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (SMB) and (iii) the difference between the return on a portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to- market stocks (HML). The model is as follows:

$$(R_{pt}) = R_f + ß_p[(R_{mt}) - R_f] + s_p(SMB) + h_p(HML) + \varepsilon_{pt} \qquad (4)$$

where:(R_{pt}) is the weighted return on portfolio p in period t.

R_f is the risk-free rate;

$ß_p$ is the coefficient loading for the excess return of the market portfolio

over the risk-free rate;

s_p is the coefficient loading for the excess average return of portfolios with small equity class over portfolios of big equity class.

h_p is the coefficient loading for the excess average returns of portfolios with high book-to-market equity class over those with low book-to-market equity class.

ε_{pt} is the error term for portfolio p at time t.

However, Kothari et al. (1995) and MacKinlay (1995) argue that a substantial part of the premium is due to 'survivor bias' and data snooping. But a number of papers have weakened and even dismissed the survivorship-bias and the data snooping hypothesis. For instance, Lakonishok et al. (1994) find a strong positive relation between average return and BE/ME for the largest 20 per cent of NYSE-Amex stocks, where survivor bias is not an issue. Similarly, Fama and French (1993) find that the relation between BE/ME and average return is strong for value-weight portfolios. As value-weight portfolios give most weight to larger stocks, any survivor bias in these portfolios is trivial. There are also are many studies using different sample periods on US and non-US data confirming the existence of the size and book-to-market equity effects.

Fama and French (1998) provide additional valuable out-of-sample evidence. They tested the FF three-factor model in thirteen different markets over the period 1975 to 1995. They find that twelve of the thirteen markets record a premium of at least

7.68 percent per annum to value stocks. Seven markets show statistically significant BM/ME betas.

Maroney and Protopapadakis (2002) tested the FF three-factor model on stock exchanges of Australia, Canada, Germany, France, Japan, the UK and the US. The size effect and the value premium survive for all the countries examined. They conclude that the size and BE/ME effects are international in character. Using a Stochastic Discount Factor (SDF) model, and a variety of macroeconomic and financial variables, do not price assets better than the Fama and French three-factor model.

Faff (2001) use Australian data over the period January 1991 to April 1999 to examine the power of the Fama French three-factor model. He finds strong support for the Fama and French three factor model, but find a significant negative rather than the expected positive, premium to small size stocks. Faff conjectures that his results are consistent with evidence from other markets, on a reversal of the size effect.

Gaunt (2004) studies the Fama French (FF) three-factor model on the Australian Stock Exchange (ASX) for a sample of 6,814 companies over the period January 1993 to December 2001. He finds that beta risk tends to be greater for smaller companies and those with lower BM ratios. However, the study does not find a strong small firm effect but there is evidence of the BM/ME effect increasing

monotonically from the lowest to the highest book-to-market equity portfolios. Overall, the evidence indicates that the three-factor model provides a better explanation of observed Australian stock returns than the CAPM.

Drew and Veeraraghavan (2003) compare the explanatory power of the single index model with the multifactor asset pricing model of Fama and French (1993) for Hong Kong, Korea, Malaysia and the Philippines. They find that small and high book-to-market equity firms generate higher returns than big and low book-to-market equity firms and conclude that the size effect and the value premium are present in these markets. They also find that the FF three-factor model explains the variation in returns better than the single index model. They suggest that the premium is a compensation for risk that is not captured by the CAPM.

There is a lack of empirical evidence of whether the size and boo-to-market equity effects are present in emerging equity markets generally, and particularly in the emerging African stock markets. This study provides some empirical evidence in an emerging market and offers additional out of sample evidence that the size and the book-to-equity effects are international in character. It moreover, augments the Fama and French three-factor model by taking into account the time variation in systematic risk.

3.0 Objectives of the Study

1. To investigate the existence of the size and book-to-market equity effects on the Stock Exchange of Mauritius (SEM).
2. To attempt an augmentation of the Fama and French (1993) three-factor model, by taking into account the time variation in betas.

3.1 Hypotheses

1. There is a size effect and a book-to-market equity effect on the SEM. In other words, there is support for the Fama and French (1993) three-factor model.
2. When the time variation in betas, are taken into account, both the size effect and the book-to-market equity effect become statistically insignificant in the Fama and French model.

3.2 Data collection

The share price and market index data for the study have been obtained from the Stock Exchange of Mauritius. However, the data was not in a form suitable for empirical analysis. So the database had to be prepared from scratch. Various issues of the SEM FactBooks were used for descriptive statistics about the market in general. Companies' annual reports' were obtained from the listed companies for the years 1998 to 2004.

3.3 Methodology

(i) Estimation of conditional market volatility

The conditional market volatility ($\sigma^2_{m,t}$) is estimated using an MA (1)-GARCH (1,1) model following the approach of Koutmos, Lee and Theodossiou (1994). Numerous studies have shown the robustness of the GARCH (1,1) as a model of stock returns [for instance, see Bollerslev et al. (1992)].

Our GARCH specification for the SEMDEX monthly returns is shown below:

$$R_{mt} = \mu + \varepsilon_{mt} + \theta\varepsilon_{mt-1} \qquad (5)$$

$$\sigma^2_{mt} = a_m + b_m\varepsilon^2_{mt-1} + c_m\sigma^2_{mt-1} \qquad (6)$$

where ε_{mt} is distributed $N(0, \sigma^2_{mt})$.

R_{mt} is the monthly market return (Semdex Return) and σ^2_{mt} is the conditional variance of the Semdex return. Equation 5 is the conditional mean of the SEMDEX return, which is modeled as an MA (1) to account for the first-order serial correlation in market returns partly induced by non-synchronous trading. Equation 6 specifies the conditional variance as a linear function of past squared residual (ε^2_{mt-1}) and past conditional variance (σ^2_{mt-1}).

Diagnostic tests will be performed to see whether the GARCH model is well specified. Only then can we use the conditional variance series in the SS market model.

(ii) Size and book to market equity effects

The methodology used by Fama and French (1993) and others requires that the stocks be split into classes according to size and book-to-market equity ratio. The median size of the whole sample is used as the breakpoint to establish the difference between the two classes. Firms with market equity less than the median value of all firms' market equity are considered as small market equity firms and those with values greater than the median value are considered as big market equity firms.

Fama and French classified the stocks into three groups of portfolios; one of low book-to-market equity (BE/ME) ratio, one of medium BE/ME ratio and the last being of high BE/ME ratio. The split of the stocks into different categories was arbitrary and Fama and French argued that there was no reason that tests should be sensitive to this choice. Following this argument and given our small sample size, only two classes of book equity-to-market equity value (low BE/ME and high BE/ME, based on the median value) will be created.

Using this type of classification, it is possible to construct four portfolios that is: H/S (High book/Small market capitalisation), H/B (High book/Big market capitalisation), L/S (Low book/Small market capitalisation) and L/B (Low book/Big market capitalisation). Value-weighted monthly returns are then calculated for each portfolio for each month from January to December over the period 1998 to 2004. The risk free rate is proxied by using the weighted monthly Treasury bill rate.

(iii) An Augmented Fama and French model

A common explanation in the literature is that the size and book-to-market equity may proxy for other risk factors not being taken into account by the Capital Asset Pricing Model. One may expect that a Fama and French three-factor that takes into account the time-variation in betas, the significance of the size and book-to-market equity effects may be reduced or disappear as the time-varying risk premium is adjusting for the temporal variation in systematic risk. The results will also confirm whether or not the FF model is robust after taking into account the time-variation in betas.

The augmented model is as follows:

$$(R_{pt}) = R_f + ß_p[(R_{m,t}) - R_f] + s_p(SMB) + h_p(HML) + \delta_p\left(\frac{R_{m,t}}{\sigma^2_{m,t}}\right) + \varepsilon_{pt} \qquad (7)$$

This regression will be performed for the four different portfolios, that is, L/S, H/S, L/B and H/B. δ_p is capturing the time-variation in beta and we expect that by taking into account time-variation in beta, s_p *and* h_p might no longer be statistically significant and that for the big market capitalization portfolios, δ_p to be positive and negative for the 'small market-cap' portfolios.

The Ordinary Least Squares method is used for the econometric analysis. The regressions showing serial correlation were corrected using the Cochrane-Orcutt

procedure. Those showing heteroscedasticity were corrected using the White's heteroscedasticity consistent variances and standard errors.

4.0 Analysis of Results

Section 4.1 presents descriptive statistics on the portfolios constructed. Next, the results for the Fama and French three-factor model are presented and discussed and finally, we consider the results for the augmented model.

4.1 Portfolio Return Characteristics

In Table 2 we report the average excess return on the four size to book-to-market equity portfolios for the Stock Exchange of Mauritius for the period January 1998 to December 2004. The table shows that the excess return on the market portfolio generates a monthly return of 0.5481. The table also shows that the H/S portfolio generates a higher return than the big stock portfolios. The two small stock portfolios generate a combined return of 2.393 per cent per month while the two big stock portfolios generate a combined return of 1.6982 per cent per month. We note that the sorted portfolios (except L / B) generate a higher average excess return than the market.

Table 2 Portfolio Average Excess Return		
Mean Monthly Return Period Jan 1998 to Dec 2004		
Portfolio	**Mean***	**Market**
L / S	1.0251 (3.3056)	0.54808 (3.4131)
H / S	1.3679 (3.909)	0.54808 (3.4131)
H / B	1.2939 (4.223)	0.54808 (3.4131)
L / B	0.40430 (2.896)	0.54808 (3.4131)

(Source: Author's computations) * standard deviation in parentheses

4.2 Estimation of the MA (1)-GARCH (1,1) model for the SEMDEX

We first estimate the MA (1)-GARCH (1,1) and conduct several specification tests to see whether the chosen GARCH model is properly specified. Only then we generate the conditional variance series of the market return, which we use in the SS market model. In fact, a Wald test on the coefficients of the variance equation shows that the null hypothesis of the sum of the coefficients is one is strongly rejected (p-value 0.000). Moreover, an ARCH test of the residuals, show no arch effects in the residuals (p-value 0.7936). All the tests confirm that the MA (1)-GARCH (1,1) is well specified and fits the monthly Semdex return series data well.

4.3 Size and Book-to- Market Equity Effects

Table 3 Results for the Fama and French Three Factor Model on the Stock Exchange of Mauritius

Model: $(R_{p,t}) - Rf = \alpha_{p,t} + ß_p(Rmt - Rf) + s_p(SMB) + h_p(HML) + \varepsilon_p t$

Portfolios excess returns	α coefficient	β coefficient	*s* coefficient	*h* coefficient	R-Bar Squared	DW-Stat
L/S	-0.5511E-4	0.59329	0.62728	-0.24452	.51408	1.9723
t-ratio[p-value]	-.017795[.986]	7.7359[.000]	7.0603[.000]	-2.9572[.004]		
L/B	-0.0014918	0.74881	-0.17156	-0.21849	.74156	2.0943
t-ratio[p-value]	-.74984[.456]	12.2351[.000]	-2.4750[.015]	-3.3081[.001]		
H/S	-0.0014918	0.74881	0.82844	0.78151	.83145	2.0943
t-ratio[p-value]	-0.74984[.456]	12.2351[.000]	11.9518[.000]	11.8326[.000]		
H/B	-0.5511E-4	0.59329	-0.37272	0.75548	.75162	1.9723
t-ratio[p-value]	-.017795[.986]	7.7359[.000]	-4.1951[.000]	9.1371[.000]		

(Source: Computed by Author)

Key: L/S = low book to equity and small market cap; H/S = high book to equity and small market cap; L/B= low book to equity and big market capitalisation and H/B=high book to equity and big market capitalisation.

Table 3 above shows the results for the Fama and French (1993) three-factor model for the Stock Exchange of Mauritius. Beta is significant for all the portfolios, but less than one. This is consistent with Gaunt (2004). The signs of the coefficients for all the portfolios are as expected and statistically significant at the one percent level. The s coefficient is positive for all the small market equity portfolios (L/S and H/S) and becomes negative for all the high market capitalization portfolios (L/B and H/B), thus confirming the existence of the small firm effect. Similarly, the h coefficient is negative for the low-book-to-equity portfolios (L/S and L/B) and becomes positive for the high book-to-equity portfolios. The SEM also confirms the existence of the book-to-market equity effect. The adjusted R^2 ranges from 51.4% to 83.1%. Our findings are consistent with those of Fama and French (1993), Drew and Veeraraghavan (2003) and others who observe that small and high book-to-market equity firms have positive slopes on SMB and HML whereas big and low book-to-market-equity firms load negatively on SMB and HML. Small firms and firms with high book-to-market equity on average earn higher returns.

4.4 An Augmented Fama and French Model

Table 4 Results of the Augmented Fama and French Three Factor Model on the Stock Exchange of Mauritius

Model: $(R_{p,t}) - Rf = \alpha_{p,t} + \beta_p(Rmt - Rf) + s_p(SMB) + h_p(HML) + \delta_p\left(\frac{R_{m,t}}{\sigma^2_{m,t}}\right) + \varepsilon_p t$

Portfolios excess returns	α coefficient	β coefficient	*s* coefficient	*h* coefficient	δ coefficient	R-Bar Squared	DW
L/S	0.010901	1.7883	0.60310	-0.25145	-0.0020749	.53083	1.9932
t-ratio[p-value]	1.4966[.138]	2.3991[.019]	6.8104[.000]	-3.1405[.002]	-1.7036[.092]		
L/B	0.010848	2.1070	-0.21343	-0.20322	-0.0022533	.75527	2.0790
t-ratio[p-value]	1.9322[.057]	3.6135[.001]	-3.0586[.003]	-3.1457[.002]	-2.3415[.022]		
H/S	0.010848	2.1070	0.78657	0.79678	-0.0022533	.84040	2.0790
t-ratio[p-value]	1.9322[.057]	3.6135[.001]	11.2718[.000]	12.3338[.000]	-2.3415[.022]		
H/B	0.010901	1.7883	-0.39690	0.74855	-0.0020749	.75581	1.9932
t-ratio[p-value]	1.4966[.138]	2.3991[.019]	-4.4819[.000]	9.3489[.000]	-1.7036[.092]		

Key: L/S = low book to equity and small market cap; H/S = high book to equity and small market cap; L/B= low book to equity and big market capitalisation and H/B=high book to equity and big market capitalisation.
(Source: Author's Computations)

We find from table 4 that for all the portfolios, the δ coefficient is significant at the 10 per cent level or better. This shows that the time variation in betas is priced. However, they are all negative in sign. Yet we expected it to negative for the small market capitalisation portfolios and positive for the big market cap portfolios. Grieb and Reyes (2001) find that out of the 38 stocks on the Brazilian stock market, 32 had negative δs'. They concluded that the Brazilian stock market behaved very much like a small capitalization market. The above results therefore concur with the fact that relatively speaking the Stock Exchange of Mauritius can be considered as a small market capitalization index and therefore we should expect most of the δ's to be negative, which is in fact the case.

The coefficients for the size effect and the book-to-market equity effect are all significant at the one percent level and with the expected signs. These effects do not disappear. This shows that the Fama and French three factor model is robust to taking into account time-varying betas. They are therefore capturing other risk factors, which are ignored by the simple CAPM model. The adjusted R^2 ranges from 53.08 to 84.04 per cent. However, the results must be interpreted with caution, as they might be sample specific. This model must be tested across other stock exchanges to test the robustness of the findings. However, to say the least, the above is a very interesting result indeed.

5.0 Conclusion

The main findings can be summarized as follows. The Fama and French three factor model holds for the Stock Exchange of Mauritius. In other words, both a size effect and a book-to-market equity are present on the SEM. The augmented Fama and French model shows that the time variation in betas is priced, but the size and book-to-market equity effects are still statistically significant. The Fama and French model is therefore robust after taking into account the time-variation in beta. An augmented Fama and French three-factor model for the SEM shows that the time-variation in beta is priced. However, it must be cautioned that this result might be sample specific. The test must be extended across other stock exchanges.

References

Bollerslev, T; R.Y. Chou, and K. F. Kroner (1992) ARCH Modeling in Finance: A Review of the Theory and Empirical Evidence, Journal of Econometrics 52, 5-59.

Drew, M.E and M. Veeraraghan (2003) Beta, Firm Size, Book-to-Market Equity and Stock Returns, Journal of the Asia Pacific Economy 8(3), 354-379.

Faff, R. (2001) An Examination of the Fama and French three-factor model using commercially available factors, Australian Journal of Management 26, 1-17.

Fama, E. F. and K.R. French (1992) The Cross Section of Expected Stock Returns, Journal of Finance 47, 427-465.

Fama, E. F. and K.R. French (1993) Common risk factors in the returns on stocks and bonds, Journal of Financial Economics 33, 3-56.

Fama, E. F. and K.R. French (1998) Value versus growth: the international evidence, Journal of Finance 53, 1975-1979.

Gaunt, C. (2004) Size and Book to Market Effects and the Fama-French three factor Asset Pricing Model: Evidence from the Australian Stock-market, Accounting and Finance 44, 27-44.

Grieb, T. and M.G. Reyes (2001) Time-Varying Betas in an Emerging Stock Market: The Case of Brazil, American Business Review, 118-124.

Kothari, S. P; J. Shanken and R.G. Sloan (1995) Another look at the cross-section of expected returns, Journal of Finance 50, 185-224.

Koutmos, G; U, Lee and P. Thedossiou (1994) Time-Varying Betas and Volatility Persistence in international Stock Markets, journal of economics and Business 46, 101-112.

Lakonishok, J; A. Shleifer and R.W. Vishny (1994) Contrarian investment, extrapolation and risk, Journal of Finance 49, 1541-1578.

Maroney, N. and A. Protopapadakis (2002) The Book-to-Market and Size Effects in a General Asset Pricing Model: Evidence from Seven National Markets, European Finance Review 6, 189-221.

MacKinlay, A. C. (1995) Multifactor models do not explain deviations from the CAPM, Journal of Financial Economics 38, 3-28.

Schwert,G.W. and P.J. Seguin (1990) Heteroscedasticity in Stock Returns, Journal of Finance 35, 915-919.

Stock Exchange of Mauritius Factbooks, various issues.