

# Multi-Year Dynamics for Forecasting Economic and Regulatory Capital in Banking

Daniel Rösch<sup>\*</sup> and Harald Scheule<sup>†‡</sup>

Key Words: Business Cycle, Correlation, Credit Risk, Basel II, Point-in-Time,  
Portfolio, Through-the-Cycle

JEL Classification: G20, G28, C51

---

<sup>\*</sup> Institute of Banking & Finance, School of Business & Economics, Leibniz University, 30167 Hannover, Germany, Phone: +49-511-762-4668, Fax : +49-511-762-4668

Email: [Daniel.Roesch@finance.uni-hannover.de](mailto:Daniel.Roesch@finance.uni-hannover.de)

<sup>†</sup> Department of Finance, The University of Melbourne, VIC 3010, Australia  
Phone: +61-3-8344-9078, Fax : +61-3-8344-6914

Email: [hscheule@unimelb.edu.au](mailto:hscheule@unimelb.edu.au)

<sup>‡</sup> The authors gratefully acknowledge the financial support of the Melbourne Centre for Financial Studies as well as the Australian Prudential Regulation Authority and thank an anonymous referee for helpful comments.

# **Multi-Year Dynamics for Forecasting Economic and Regulatory Capital in Banking**

## **Abstract**

The determination of future credit loss distributions constitutes a fundamental challenge in many credit risk applications such as the calculation of economic and regulatory capital as well as the pricing of loans, portfolios or derivatives thereof. Currently, best practice is to assume a one-year risk horizon for the derivation of the credit loss distribution. However, the maturities of most credit risky products exceed one year and the credit loss of the whole product life has to be taken into account. The present article investigates the impact of multi-year forecasts of credit risk parameters such as probabilities of default and correlations on the distribution of future losses to a credit portfolio. Moreover, the implications are demonstrated for collateralized debt obligations.

# 1 Introduction

It is well known that the banking industry experiences losses in relation to their credit exposures of varying magnitudes over time. For example, the years 2001 and 2002 with bankruptcies including Enron and WorldCom saw tremendous credit losses to global financial institutions. The number of credit defaults of rated companies during these years was nearly six times as high as in 2005. These credit loss cycles are often described by either correlations, i.e., dependencies between companies or alternatively by proxies of the business cycle.

This article is particularly motivated by the industry's need to forecast the distribution of future losses to a credit portfolio over multiple years. Most current models focus on a limited set of parameters, such as the probability of default, asset correlation, loss given default or exposure at default for a one-year risk horizon. These measures are developed and implemented in line with the proposal of a new capital adequacy framework for banks by the Basel Committee on Banking Supervision (2004), better known as 'Basel II'.

Previous research has shown that asset correlations depend on the chosen modelling methodology for the default probabilities (see Hamerle et al., 2006, Rösch, 2005 and Rösch et al., 2004 and 2005). Therefore, the present contribution investigates how different modelling methodologies, such as the point-in-time and through-the-cycle approach, affect correlation estimates for multiple years. A non-linear framework is developed to simultaneously forecast default probabilities and asset correlations for multiple years. This framework allows the analysis of the impact of different modelling approaches and different time horizons on the distribution of future credit portfolio losses to a financial institution. Specifically, the hypothesis is that as the maturity of credit risk products increases, the business cycle becomes less important and dilutes the

importance of point-in-time models. In addition, the behaviour of asset correlations in dependence on the maturity is analyzed.

The findings are expected to be highly relevant to the industry, in particular for modelling credit risky products with maturities greater than one year. The developed framework may be used in many applications; among these is the pricing of collateralized debt obligations, i.e., securitized investment tranches of loan portfolios with different seniorities.

In the existing literature, Duffie et al. (2007) are the first-in-kind to model default probabilities by observable observation for multiple years. Previously, researchers focused their models for default probabilities on one-period time horizons (e.g., Shumway, 2001, Hamerle et al., 2006, Hillegeist et al., 2004 or McNeil et al. 2007).

The present paper extends this literature by developing a framework which allows the simultaneous estimation of default probabilities and asset correlations for multiple years based on information which is observable at the point-in-time at which the forecast is made. As a result, correlations will increase over time because the level of macroeconomic information known today about future years decreases. In contrast to these findings, many of today's applications such as the pricing of CDOs rely on constant, i.e., time invariant, values for correlations. Examples are the plain assumption of constant values by practitioners in credit portfolio models (e.g., Fitch Ratings, 2006; Moody's, 2006; Standard & Poor's, 2005), the derivation of constant values from quotes for standard tranches of CDS indices (e.g., Hull et al., 2004 or Anderson et al., 2004) and the restriction of studies to single-year time horizons (see the above mentioned literature). Note that Duffie et al. (2007) model the association between default probabilities explained by systematic risk drivers but not any unobservable co-movement captured by the correlation estimates in modern credit risk applications.

The article is organized into four sections. Following the introduction, the second section develops a theoretical framework to describe the process of credit default and two fundamental methodologies to estimate the parameters for observable variables driving this process. The third section includes an empirical analysis of the default history of rated companies and the fourth concludes with a discussion.

## 2 Multi-year dynamics

### 2.1 Process for the credit quality

The event in which an obligor misses a payment obligation is defined as a default. The observable default event for obligor  $i$  in time period  $t$  is random and modelled using the indicator variable  $Y_{it}$  :

$$Y_{it} = \begin{cases} 1 & \text{obligor } i \text{ defaults in period } t \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

( $i \in N_t; t = 1, \dots, T$ ).  $N_t$  denotes the set of all obligors who did not default at the beginning of time period  $t$ . The time period  $[1, T]$  is called the estimation period.

In addition, the continuous non-observable variable  $R_{it}$  is defined, which may be interpreted as the logarithmic return on an obligor's assets. A threshold-value model is assumed for the relationship between the return on assets and the default event  $Y_{it}$ . Default is equivalent to the return on assets falling below the threshold  $c_{it}$ , i.e.,

$$R_{it} \leq c_{it} \Leftrightarrow Y_{it} = 1 \quad (2)$$

For the continuous non-observable variable  $R_{it}$  we assume the simple model

$$R_{it} = -wF_t + \sqrt{1-w^2}U_{it}, \quad (3)$$

where  $F_t$  denotes a common systematic risk factor, and  $U_{it}$  denotes an obligor-specific idiosyncratic risk factor. Throughout this paper, we assume that  $F_t$  and  $U_{it}$  are i.i.d. standard normally distributed random variables. Hamerle et al. (2006) show that other distributions such as the logistic distribution can alternatively be chosen. Note that model (3) is assumed by the Basel Committee on Banking Supervision (2004) in its Internal-Rating-Based approach in order to calculate the regulatory capital of banks (see Finger, 2001).

Furthermore, we include time-lagged observable risk drivers into the model by assuming the following linear specification for the threshold  $c$  :

$$c_t = \alpha + \beta z_{t-1} \quad (4)$$

where  $z_{t-1}$  denotes a time-lagged systematic risk factor, like the unemployment rate or the interest rate. The time-lagged risk factors are known at the point in time at which the forecast is made. In the empirical section we will analyze the thresholds for different

risk segments, defined by the Standard & Poor's rating classes  $j$  with  $j \in \{'AAA \text{ to } BBB', 'BB', 'B', 'CCC \text{ to } C'\}$ :

$$c_t^j = \alpha^j + \beta^j z_{t-1}^j \quad (5)$$

Extensions could incorporate multiple time-varying and time-lagged obligor-specific ( $\mathbf{x}_{it-1}$ ) and systematic risk factors ( $\mathbf{z}_{t-1}$ ) into the model (e.g., Hamerle et al., 2006):

$$c_{it} = \alpha + \beta_x' \mathbf{x}_{it-1} + \beta_z' \mathbf{z}_{t-1}. \quad (6)$$

However, these extensions are not part of this contribution due to data constraints.

## 2.2 Estimation methodologies

The credit risk community broadly differentiates between two fundamental modelling methodologies:

- Point-in-time (PIT) models base probabilities of default on the current state of the business cycle which in the following, will be approximated by macroeconomic variables;
- Through-the-cycle (TTC) models base probabilities of default on the average state of the business cycle. In the following, default probabilities will equal the historic average of default rates for a given rating class  $j$ .

Within the bespoke community, a discussion on the correct definition of a through-the-cycle and point-in-time model exists and it should be noted that there is no universal agreement to date. We use these expressions as highly stylized denominations, being aware that other interpretations of these rating philosophies may exist (see Heitfield, 2005 for a discussion).

One obtains the conditional point-in-time default probability, i.e., the conditional probability of default given the realization of the unobservable systematic factor  $f_t$ :

$$\begin{aligned}
CPD_{it}^{PIT}(z_{t-1}, f_t) &= P(Y_{it} = 1 | z_{t-1}, f_t) \\
&= P(R_{it} \leq c_t | z_{t-1}, f_t) \\
&= P(\sqrt{1-w^2}U_{it} \leq \alpha + \beta z_{t-1} + wf_t) \\
&= \Phi\left(\frac{\alpha + \beta z_{t-1} + wf_t}{\sqrt{1-w^2}}\right)
\end{aligned} \tag{7}$$

and for the unconditional point-in-time default probability:

$$\begin{aligned}
PD_{it}^{PIT}(z_{t-1}) &= P(Y_{it} = 1 | z_{t-1}) \\
&= P(R_{it} \leq c_t | z_{t-1}) \\
&= P(\sqrt{1-w^2}U_{it} - wF_t \leq \alpha + \beta z_{t-1}) \\
&= \Phi(\alpha + \beta z_{t-1})
\end{aligned} \tag{8}$$

since the standard deviation of  $\sqrt{1-w^2}U_{it} - wF_t$  equals one.

In a through-the-cycle model,  $Z_{t-1}$  may be assumed to be unknown and normally distributed. Thus, the systematic random variables  $Z_{t-1}$  and  $F_t$  may be modelled by one

random variable  $F_t^*$  which is standard normally distributed. One then obtains for the conditional through-the-cycle default probability:

$$\begin{aligned}
CPD_{it}^{TTC}(f_t^*) &= P(Y_{it} = 1 | f_t^*) \\
&= P(R_{it} \leq c_t | f_t^*) \\
&= P(\sqrt{1-w^2}U_{it} \leq \alpha + \nu \cdot f_t^*) \\
&= \Phi\left(\frac{\alpha + \nu \cdot f_t^*}{\sqrt{1-w^2}}\right)
\end{aligned} \tag{9}$$

with  $\nu = \sqrt{\beta^2 \text{Var}(Z_{t-1}) + w^2}$  and for the unconditional through-the-cycle default probability:

$$\begin{aligned}
PD_{it}^{TTC} &= P(Y_{it} = 1) \\
&= P(R_{it} \leq c_t) \\
&= P(\sqrt{1-w^2}U_{it} - \sqrt{\beta^2 \text{Var}(Z_{t-1}) + w^2} \cdot F_t^* \leq \alpha) = . \\
&= \Phi\left(\frac{\alpha}{\sqrt{1 + \beta^2 \text{Var}(Z_{t-1})}}\right)
\end{aligned} \tag{10}$$

since the standard deviation from  $\sqrt{1-w^2}U_{it} - \sqrt{\beta^2 \text{Var}(Z_{t-1}) + w^2} \cdot F_t^*$  equals  $\sqrt{1 + \beta^2 \text{Var}(Z_{t-1})}$ . Note that the parameter  $w$  describes the co-movement of the probabilities for given time periods from which the asset correlation (i.e., the correlation between asset returns) can be derived for the point-in-time model

$$Corr^{PII}(R_{it}, R_{jt}) = w^2, \quad (11)$$

as well as the through-the-cycle model:

$$Corr^{TTC}(R_{it}, R_{jt}) = \frac{w^2 + \beta^2 \text{Var}(Z_{t-1})}{1 + \beta^2 \text{Var}(Z_{t-1})}, \quad (12)$$

Given the availability of historic default data, the parameters can be estimated by maximizing the log-likelihood (with respect to  $F_t$  or  $F_t^*$  marginal) over all obligors and periods:

$$\ln L = \sum_{t=1}^T \ln \int_{-\infty}^{\infty} \left[ \prod_{i \in N_t} \left[ CPD_{it}^{PII}(f_t)^{y_{it}} (1 - CPD_{it}^{PII}(f_t))^{(1-y_{it})} \right] \varphi(f_t) \right] df_t \quad \text{or} \quad (13)$$

$$\ln L = \sum_{t=1}^T \ln \int_{-\infty}^{\infty} \left[ \prod_{i \in N_t} \left[ CPD_{it}^{TTC}(f_t^*)^{y_{it}} (1 - CPD_{it}^{TTC}(f_t^*))^{(1-y_{it})} \right] \varphi(f_t^*) \right] df_t^* . \quad (14)$$

Note that  $\varphi(f_t) = (1/\sqrt{2\pi}) \exp(-0.5f_t^2)$  denotes the density function of the standard normal distribution. Both equations contain  $T$  integrals which can be solved approximately using adaptive Gauss-Hermite-quadrature (Pinheiro et al., 1995 or Rabe-Hesketh et al., 2002, pp. 5-9). It follows from the general theory of Maximum-Likelihood estimation that the estimates exist asymptotically and are consistent as well as asymptotically normally distributed (see Davidson et al., 1993, pp. 243 et seq.).

### 2.3 Forecasting obligor credit risk for single and multiple years

Thus far we have presented the models for probabilities of default based on historic default observations and time-lagged macroeconomic risk drivers, where a time lag of one year was used.

As a matter of fact, the industry frequently focuses on multi-year credit risk forecasts using the through-the-cycle model and rating data provided by the rating agencies. In extensions, this data is often modified by internal forecasts or particularities of the respective portfolios such as expected loan losses or changes in composition. In one approach, cumulative default rates are calculated as the ratio of the number of defaulted obligors to the total number of obligors for a static portfolio of obligors. The next figure shows the cumulative default rates for different rating classes published by Standard and Poor's (2005).

[Insert Figure 1 here]

Cumulative default rates which are used as estimates for cumulative default probabilities, are calculated by:

$$CumDR_T = 1 - \prod_{t=1}^T (1 - DR_t) \quad (15)$$

Practitioners derive (marginal) default rates as proxies for the default probabilities of future years from the cumulative default rates:

$$DR_T = 1 - \frac{1 - CumDR_T}{\prod_{t=1}^{T-1} (1 - DR_t)} \quad (16)$$

Apart from this through-the-cycle approach to calculate historic multi-year default probabilities little research has focused on i) point-in-time forecasts of default probabilities for ii) multiple future time periods.

The through-the-cycle model (10) can be easily used for forecasting the risk of future years. However, if we assume that the point-in-time model (7) is based on risk drivers which are lagged by one year, these risk drivers have not been realized by the time the forecast is made. Therefore, they are unknown for periods beyond one year into the future. For ease of exposition we assume a simple AR(1) process for  $Z_t$ :

$$Z_t = \gamma z_{t-1} + \sigma \varepsilon_t \quad (17)$$

Note that in the empirical analysis we use the deviation of a macroeconomic risk driver from its mean. Hence, we did not include an intercept in order to minimize the number of parameters to be estimated. The process assumes a constant volatility. The implications of more complicated processes may be studied in future extensions.

We assume that the realization of the macroeconomic risk driver in the final year of our estimation period is known ( $z_T$ ) and the risk driver for the following year  $T+1$  ( $Z_{T+1}$ ) will be simulated using model (18). The change from a deterministic to a random risk driver changes equation (7) in the following year ( $T+2$ ) to:

$$\begin{aligned}
CPD_{iT+2}^{PIT} &= P\left(\sqrt{1-w^2}U_{iT+2} \leq \alpha + \beta(\gamma z_T + \sigma\varepsilon_{T+1}) + wf_{T+2}\right) \\
&= P\left(\sqrt{1-w^2}U_{iT+2} \leq \alpha + \beta\gamma z_T + \beta\sigma\varepsilon_{T+1} + wf_{T+2}\right) \\
&= \Phi\left(\frac{\alpha + \beta\gamma z_T + \beta\sigma\varepsilon_{T+1} + wf_{T+2}}{\sqrt{1-w^2}}\right)
\end{aligned} \tag{18}$$

and equation (8) to:

$$PD_{iT+2}^{PIT} = \Phi\left(\frac{\alpha + \beta\gamma z_T}{\sqrt{1 + \beta^2\sigma^2}}\right) . \tag{19}$$

In this model, the variance of the macroeconomic risk driver increases every year:

$$Var(Z_{T+\tau}) = \beta^2\sigma^2 \sum_{\substack{s=1 \\ \tau \geq 2}}^{\tau-1} \gamma^{2(s-1)} . \tag{20}$$

with  $\tau = 2, \dots, M$ . The time period  $[T+1, \dots, T+M]$  is called the forecast period. Note that  $Var(Z_T) = 0$  since we assume that the macroeconomic risk factor is known in the first period and random thereafter. Table 1 shows the variance of the macroeconomic risk driver for the first four periods:

[Insert Table 1 here]

Therefore, the point-in-time model for forecasting can generally be written as:

$$CPD_{iT+\tau}^{PIT} = \Phi \left( \frac{\alpha + \beta \gamma^{(\tau-1)} z_T + \beta \sigma \sum_{\substack{s=1 \\ \tau \geq 2}}^{\tau-1} (\gamma^{(s-1)} \varepsilon_{T+\tau-1}) + w f_{T+\tau}}{\sqrt{1-w^2}} \right) \text{ and} \quad (21)$$

$$PD_{iT+\tau}^{PIT} = \Phi \left( \frac{\alpha + \beta \gamma^{(\tau-1)} z_T}{\sqrt{1 + \beta^2 \sigma^2 \sum_{\substack{s=1 \\ \tau \geq 2}}^{\tau-1} \gamma^{2(s-1)}}} \right). \quad (22)$$

The asset correlations are in this case:

$$Corr^{PIT} (R_u, R_j) = \frac{\beta^2 \sigma^2 \sum_{\substack{s=1 \\ \tau \geq 2}}^{\tau-1} \gamma^{2(s-1)} + w^2}{1 + \beta^2 \sigma^2 \sum_{\substack{s=1 \\ \tau \geq 2}}^{\tau-1} \gamma^{2(s-1)}}. \quad (23)$$

The variance introduced in year  $T + \tau$  ( $\tau \geq 2$ ) by the macroeconomic factor is

$\beta^2 \sigma^2 \sum_{\substack{s=1 \\ \tau \geq 2}}^{\tau-1} \gamma^{2(s-1)}$  for the PIT model. In the TTC model we assumed that  $Z_T$  is unknown

and random. Therefore, the variance of the macroeconomic factor is  $\beta^2 \text{Var}(Z_t)$  in

every period. Figure 2 (high dependency,  $\beta^2 = 0.9$ ) and Figure 3 (low dependency,

$\beta^2 = 0.1$ ) show for different business cycle dependencies that the variance of the

macroeconomic factor in the PIT model converges to this value over time. We will see

in the empirical analysis that this uncertainty translates into higher forecast uncertainty

of the PD parameter and the derived credit losses. As a result, the economic (and regulatory) capital is expected to increase.

[Insert Figure 2 here]

[Insert Figure 3 here]

The charts also show that the variance increases with the

- Time as less information on the state of the economy is known,
- Dependence on the economy (i.e., increase of  $\beta^2$ ) and
- Decrease in forecast quality of the macroeconomic risk driver (i.e., decrease of  $\gamma^2$  and increase of  $\sigma^2$ ).

#### 2.4 Forecasting credit portfolio loss distributions (for a single year and multiple years)

Given the parameters of the models  $CPD_{iT+\tau}^{PIT}$  and  $CPD_{iT+\tau}^{TTC}$ , the default distribution, i.e., the distribution of the potential numbers of defaulting borrowers for the years  $T + \tau$  can be simulated. Following Vasicek (1987), the probability distribution for the number  $d_{T+\tau}$  of defaulting companies given  $n_{T+\tau} = n_{T+\tau-1} - d_{T+\tau-1}$  companies can be calculated for homogeneous risk segments:

$$P^{PIT}(d_{T+\tau}) = \begin{cases} \binom{n_{T+\tau}}{d_{T+\tau}} \cdot \int_{-\infty}^{+\infty} \left[ [CPD_{T+\tau}^{PIT}]^{d_{T+\tau}} \cdot [1 - CPD_{T+\tau}^{PIT}]^{n_{T+\tau} - d_{T+\tau}} \right] \varphi(f_{T+\tau}) df_{T+\tau} & \tau = 0, \dots, M; d_{T+\tau} = 0, \dots, n_{T+\tau} \\ 0 & \text{else} \end{cases}$$

or

(24)

$$P^{TTC}(d_{T+\tau}) = \begin{cases} \binom{n_{T+\tau}}{d_{T+\tau}} \cdot \int_{-\infty}^{+\infty} \left[ [CPD_{T+\tau}^{TTC}]^{d_{T+\tau}} \cdot [1 - CPD_{T+\tau}^{TTC}]^{n_{T+\tau} - d_{T+\tau}} \right] \varphi(f_{T+\tau}^*) df_{T+\tau}^* & \tau = 0, 1, 2, \dots, M; d_{T+\tau} = 0, \dots, n_{T+\tau} \\ 0 & \text{else} \end{cases} \quad (25)$$

The default distribution can be interpreted as a loss distribution by assuming values for the loss given default and the exposure at default of one.

### 3 Empirical Analysis

#### 3.1 Data description

The empirical analysis is based on a public available default data set from Standard and Poor's (2005) which includes the number of defaults and total companies for given rating classes and the years 1981 to 2005. We restrict the analysis to the years 1987 to 2005 and the ratings to the rating classes 'AAA to BBB', 'BB', 'B' and 'CCC-C' due to a limited number of defaults in initial years and for the rating classes indicating a better credit quality. Therefore, the rating class 'AAA to BBB' includes companies which are rated 'AAA', 'AA', 'A' and 'BBB'. The empirical analysis may be extended to more granular rating classes in future years. The following chart shows the default rates (using the logarithmic scale) for the four chosen categories.

[Insert Figure 4 here]

Note that Standard and Poor's records a default upon the missing of a payment in relation to any financial obligation, the announcement of a distressed exchange offer

(i.e., debt holders are coerced into accepting substitute instruments with lower coupons) or the filing of bankruptcy.

The following two tables show the number of observations and defaults for each rating category and year:

[Insert Table 2 here]

[Insert Table 3 here]

Since the majority of rated companies are U.S. companies we focus on macroeconomic risk factors of the U.S. economy. Consistent with a previous study using data from Moody's Investor Services (see Rösch et al., 2005) we chose the composite index of 10 leading indicators (LEAD) published by The Conference Board ([www.tcb-indicators.org](http://www.tcb-indicators.org)) as a macroeconomic systematic risk driver. The index measures the future health of the U.S. economy and is based on the

- Average weekly hours in manufacturing,
- Average weekly initial claims for unemployment insurance,
- Manufacturers' new orders of consumer goods and materials,
- Vendor performance,
- Manufacturer's new orders of non-defence capital goods,
- Building permits for new private housing units,
- Stock price index,
- Money supply,
- Interest rate spread of 10-year treasury bonds less federal funds and
- Consumer expectations.

The index is recognized as an indicator for the future state of the U.S. business cycle. Note that for the analysis, growth rates of the index were calculated and lagged by one year and the mean of 0.0315 subtracted.

Table 4 and Table 5 show descriptive statistics and Bravais-Pearson correlation coefficients for the default rates as well as the time-lagged variable LEAD.

[Insert Table 4 here]

[Insert Table 5 here]

### 3.2 Model estimation

Given the data we estimated a TTC and a PIT model for every rating category. Table 6 shows the estimated parameters:

[Insert Table 6 here]

The parameter estimates for the AR(1) process of the macroeconomic risk factor are:

- $\gamma = 0.2988$  with a standard error of 0.2211 and a p-value of 0.1932;
- $\sigma = 0.0287$ .

The r-square for this linear regression was 0.0922. As a result, the macroeconomic risk factor can be explained by its time lagged variable only to a limited extent. This is confirmed by Figure 5 which shows that the variance of the macroeconomic factor in the PIT model converges already in the third year to the one of the TTC model.

[Insert Figure 5 here]

Table 7 shows the asset correlations for the models for years 2006 ( $T + 1$ ), 2007 ( $T + 2$ ) and 2008 ( $T + 3$ ):

[Insert Table 7 here]

Note that the PIT-values converge to the TTC-values. Minor inconsistencies can be explained by the asymptotic property of the estimated parameters.

Consistent with previous studies (e.g., Rösch et al., 2004 and 2005), these asset correlations are lower than those proposed by the Basel Committee on Banking Supervision (2004) which are between 24% and 12% and decrease with the probability of default. In comparison to the Basel proposals and a study by Lopez (2004), our empirical analysis, which is based on default histories of rated companies does not support the inverse relationship between the asset correlation and the credit quality of companies. Please note that the standard errors for the estimates of the sensitivities of the systematic risk drivers ( $\beta$ ,  $w$  and  $\nu$ ) are relatively high due to a limited number of observation periods.

### **3.3 Forecast Loss distributions**

The loss distributions of a hypothetical portfolio of loans will be forecast based on the eight estimated models. Since we look in the next section at an application of our contribution, namely collateralized debt obligations, we assume a credit portfolio with the following characteristics:

- 125 loans with different obligors. Important CDS indices, such as the Dow Jones CDX North America Investment Grade or the iTraxx Europe index are based on 125 obligors.
- The asset return of each obligor is independent given the state of the economy.
- The loans are rated into one of our four modelled categories ‘AAA to BBB’, ‘BB’, ‘B’, or ‘CCC to C’.
- The maturity of the loans (risk horizon of the loans) equals 1 or 10 years.

- Every credit default incurs a loss given default of 45% (which is in line with the proposed values for senior secured claims in the Foundation IRB approach proposed by the Basel Committee on Banking Supervision, 2004).
- The exposure at default equals the current exposure of every loan, which is 0.8 monetary units. This number was chosen so that the total exposure equals 100 (125 obligors  $\times$  0.8) and the resulting losses can be interpreted in monetary units as well as a percentage of the total exposure.

The forecast loss distributions are simulated on the estimated parameters of Table 6 using 100,000 iterations. In addition, the loss distributions of the PIT models are based on

- An average observation for the state of the economy ( $LEAD_{2005} = 0$ ). Note that the growth rate of the composite index of 10 leading indicators in this case is 0.0315. This model will be called PIT-1 and
- The realized value  $LEAD_{2005} = -0.0111$  for 2005. Note that the growth rate of the composite index of 10 leading indicators in this case is  $-0.0111 + 0.0315 = 0.0204$ . This model will be called PIT-2.

In order to illustrate how the path of the simulated macroeconomic variable affects the loss distributions, two representative simulations were run for a ‘booming’ economy and a recession under the PIT-1 model. Figure 6 shows different paths for the macroeconomic risk drivers and Figure 7 the resulting cumulative loss distributions (assumption rating class ‘BB’ with a maturity of 10 years):

[Insert Figure 6 here]

[Insert Figure 7 here]

It can be seen that the future cumulative credit losses increase with time at a decreasing rate with the realizations of the systematic (macroeconomic) risk driver.

Table 8 (maturity of 1 year) and Table 9 (maturity of 10 years) display the characteristics of the different loss distributions of the models:

[Insert Table 8 here]

[Insert Table 9 here]

It can be seen that the mean loss (i.e., the Expected Loss, EL) increases with a lower credit quality (from ‘AAA to BBB’ to ‘CCC to C’) and a longer maturity (1 to 10 years). The dispersion is generally measured by the difference between a certain percentile of the loss distribution and the Expected Loss (i.e., the Credit Value at Risk, CVaR). While the percentiles also increase with lower credit quality and longer maturity, the CVaR shares this property only for the 1-year credit portfolio. The 10-year CVaR for the ‘CCC to C’ rated credit portfolio for instance is much lower (TTC: 1.30) than for the ‘B’ rated credit portfolio.

The Expected Losses for PIT-1 are slightly lower than for PIT-2 due to the lower starting value of the macroeconomic risk driver and convergence for longer maturities. The dispersion of credit losses is higher for the TTC model than for the PIT models for the 1-year portfolio and converges for longer maturities.

In line with the IRB approach of the proposals of the current Basel Committee on Banking Supervision [2004] the credit portfolio risk was measured with the Credit-Value-at-Risk at the 99.9<sup>th</sup> percentile, i.e., the difference between the 99.9<sup>th</sup> percentile of the loss distribution and the Expected Loss. The following table compares the Basel values (based on a one-year risk horizon) with the forecast CVaR for the first year:

[Insert Table 10 here]

Contrary to the simulated CVaR, the Basel CVaR is calculated on the following assumptions:

- The credit portfolio is not subject to idiosyncratic risk, i.e., is infinitely granular;

- Asset correlations are prescribed given the exposure class, default probabilities as well as the annual turnover (for small and medium sized enterprises) and
- Corporate loans with a longer maturity are more risky.

It can be seen that the forecast CVaR as well as the respective value derived from the Basel proposals increases for a one-year horizon with the probability of default. Due to the assumption of higher asset correlations (compare section 3.2) the Basel value is higher than the forecast for all models. Note that Table 8 to Table 9 have shown that the CVaR may decrease for high expected losses due to a lower credit quality or longer risk horizons.

### **3.4 Application for Collateralized Debt Obligations**

Given the forecast loss distributions, the losses to investors in the loan portfolio with different seniorities (tranches) can be calculated. The situation is comparable to collateralized debt obligations. In addition to the preceding chapter, we assume that the investors' exposures equal 3% (equity tranche), 4%, 3%, 5%, 15% (mezzanine tranches) and 70% (senior tranche) of the portfolio's total exposure.

The following table shows the exposure to the different investors assuming a credit portfolio with BB-rated loans with a maturity of 1 or 10 years. Looking at Table 2 we see that only 15.44% of all rated companies are 'BB' rated. However, the average historic default rate of this rating class is in line with the default rates of many small and medium sized enterprise as well as retail portfolios (in particular non-mortgage loans). Table 11 compares the PIT-1 model with the TTC model for two credit portfolios with a maturity of 1 year and 10 years.

[Insert Table 11 here]

Generally speaking, the seniority of a tranche makes its exposure to credit losses subject to the dispersion of the credit losses. The higher this dispersion is, the greater the likelihood that a senior tranche experiences a loss. For the portfolio with a maturity of 1 year, the PIT-1 leads to a lower dispersion of the credit losses than the TTC model. As a result, in the PIT-1 model all losses are covered by the first, i.e., the equity tranche, whereas in the TTC model losses are covered both the first and the second tranche. This effect is no longer observable for a 10 year maturity. However, the likelihood that senior tranches experience losses and their associated expected loss increases with the risk horizon (maturity).

## **4 Discussion**

The theoretical section along with its empirical analysis has shown that the modeling methodology has an important influence on the forecast of future credit loss distributions. A through-the-cycle rating assumes that credit events in a given year occur jointly by random. A point-in-time rating tries to identify systematic risk drivers which are responsible for this co-movement and explain future credit losses by information which is known at the time the forecast is made.

This information decreases in importance overtime and the average nominal forecast credit loss to a credit portfolio converges during the course of the business cycle for these two fundamental modelling methodologies. This may cause the impression that the choice between the two approaches is arbitrary. However, the numbers have shown that the information which is known at the time the forecast is made, (i.e., the value of the risk driver) is important for the forecast of the level and dispersion of future credit losses. In addition, it should be noted that the timing of credit losses is important due to the time value of money.

The empirical analysis was constrained by the availability of data. The data set used included the default history for a portfolio of rated but otherwise anonymous companies. Therefore, only an analysis of systematic risk drivers was possible. In addition, the number of rated companies was limited and increased from 1,833 companies in 1987 to 4,965 companies in 2005.

The present study focused on the parameters default probability and asset correlation. Other studies (e.g., Carey, 1998, Frye, 2003, Altman et al., 2006) have shown that other parameters such as exposures or losses given default also show cyclical behaviours as well.

Beyond these limitations, the study has developed a statistical framework which allows banks to model the credit risk of their risk segments and products point-in-time for multiple years. This may decrease the uncertainty of current forecasts and therefore support the future measurement and management of risks to credit portfolios in general and credit risk transfers in particular.

## 5 References

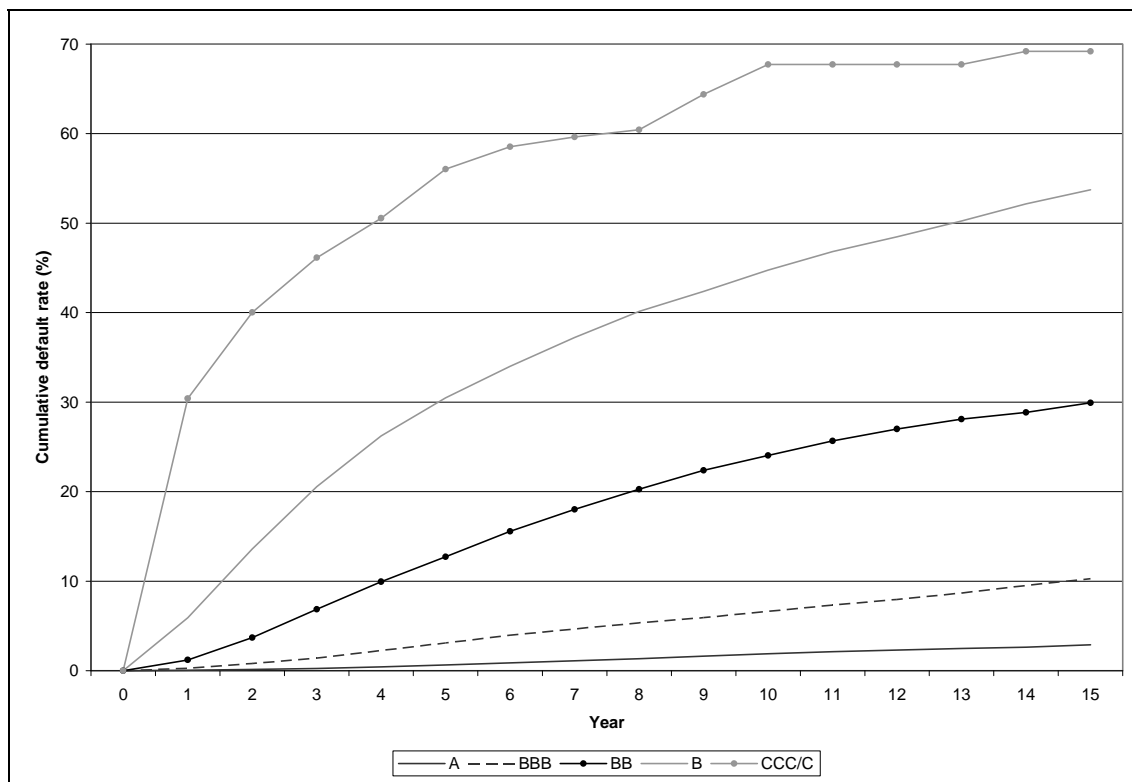
- Altman, E., Brady, B., Resti, A. & Sironi, A. (2006), 'The link between default and recovery rates: Theory, empirical evidence and implications', *Journal of Business*, 78, 2203–2227.
- Anderson, L., Sidenius, J., 2004. Extensions to the Gaussian copula: random recovery and random factor loadings, *Journal of Credit Risk*, 1, 29-70.
- Basel Committee on Banking Supervision, 2004. International Convergence of Capital Measurement and Capital Standards - A Revised Framework, Consultative Document, Bank for International Settlements, June.
- Carey, M., 1998. Credit Risk in Private Debt Portfolios, *Journal of Finance*, 53, 1363-1387.
- Davidson, R., MacKinnon, J.G., 1993. *Estimation and Inference in Econometrics*, Oxford University Press, New York.
- Duffie, D., Saita, L., Wang, K., 2007. Multi-Period Corporate Default Prediction with Stochastic Covariates, *Journal of Financial Economics*, 83, 635-665.
- Finger, C.C., 2001. The One-Factor CreditMetrics Model in the New Basel Capital Accord, *RiskMetrics Journal*, 9-18.
- Fitch Ratings, 2006. Exposure draft: Introducing the Fitch VECTOR Default Model Version 3.0'.
- Frye, J., 2003. A False Sense of Security, *Risk*, 16, 63-67.
- Hamerle, A., Rösch, D., 2006, Parameterizing Credit Risk Models, *Journal of Credit Risk* 2, No. 4, 101-122.

- Hamerle, A., Liebig, T. & Scheule, H., 2006, Forecasting credit event frequency - empirical evidence for West German firms, *Journal of Risk*, 9, 75-98.
- Heitfield, E.A., 2005. Dynamics of Rating Systems, in: Basel Committee on Banking Supervision: Studies on the Validation of Internal Rating Systems, Working Paper No. 14, February, 10-27.
- Hillegeist, S.A., Keating, E.K., Cram, D.P., Lundstedt, K.G., 2004. Assessing the Probability of Bankruptcy, *Review of Accounting Studies*, 9, 5-34.
- Hull, J., White, A., 2004. Valuation of a CDO and an n-th to default CDS without Monte Carlo simulation, *Journal of Derivatives*, 12, 8-16.
- Kalbfleisch, J.D., Prentice, R.L., 1980. *The Statistical Analysis of Failure Time Data*, New York.
- Lopez, J., 2004. The empirical relationship between average asset correlation, firm probability of default, and asset size. *Journal of Financial Intermediation*, 13, 265–283.
- McNeil, A.J. and Wendin, J.P., 2007. Bayesian inference for generalized linear mixed models of portfolio credit risk. *Journal of Empirical Finance*, 14, 131-149.
- Moody's Investors Service, 2006. CDOROMv2.3 User Guide.
- Pinheiro, J.C./Bates, D.M., 1995. Approximations to the Log-Likelihood Function in the Nonlinear Mixed-Effects Model, *Journal of Computational and Graphical Statistics*, 4, 12- 35.
- Rabe-Hesketh, S., Skrondal, A., Pickles, A., 2002. Estimation of generalized linear mixed models, *Stata Journal*, 2, 1- 21.
- Rösch, D., 2005, An Empirical Comparison of Default Risk Forecasts from Alternative Credit Rating Philosophies, *International Journal of Forecasting* 21, 37-51.

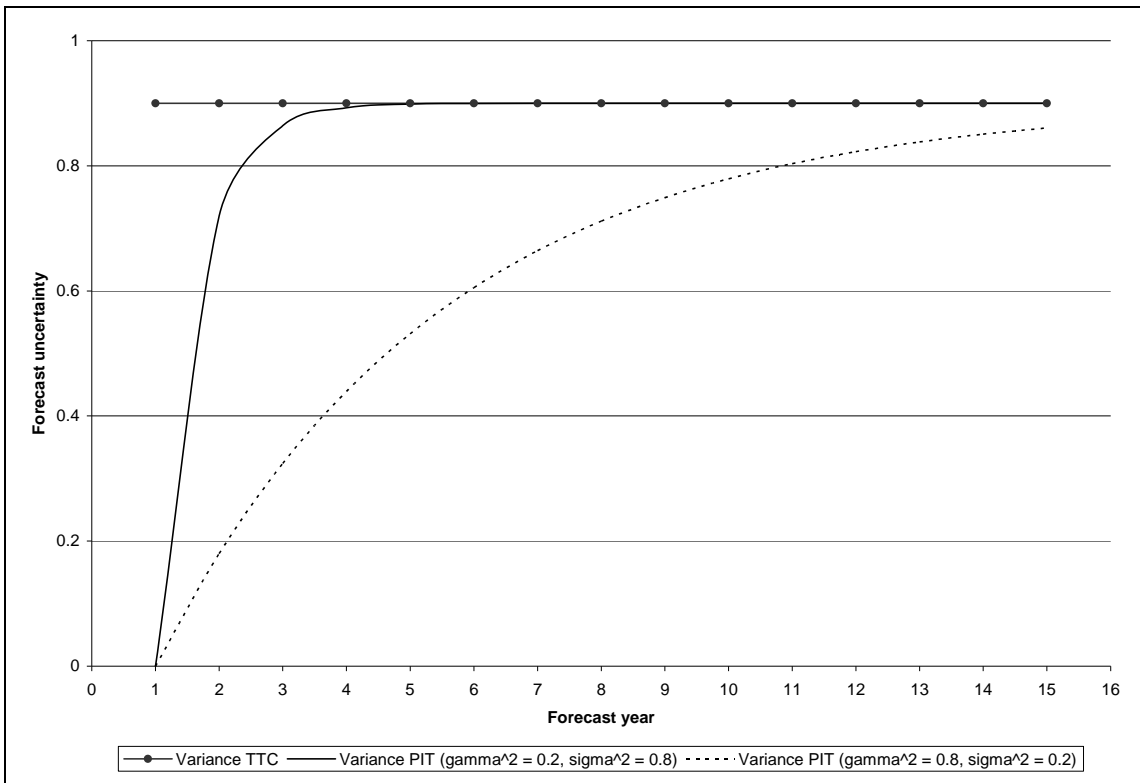
- Rösch, D., Scheule, H., 2005. A Multi-Factor Approach for Systematic Default and Recovery Risk, *Journal of Fixed Income*, 15, 63-75.
- Rösch, D., Scheule, H., 2004. Forecasting Retail Portfolio Credit Risk, *Journal of Risk Finance*, 5, 2, 16-32.
- Shumway, T., 2001. Forecasting Bankruptcy More Accurately: A Simple Hazard Model, *Journal of Business*, 74, 101- 124.
- Standard and Poor's, 2005. Annual 2005 Global Corporate Default Study and Rating Transitions.
- Standard and Poor's, 2006. CDO Evaluator Handbook.
- Vasicek, O.A., 1987. Probability of Loss on Loan Portfolio, Working Paper, KMV Corporation.

## Figures

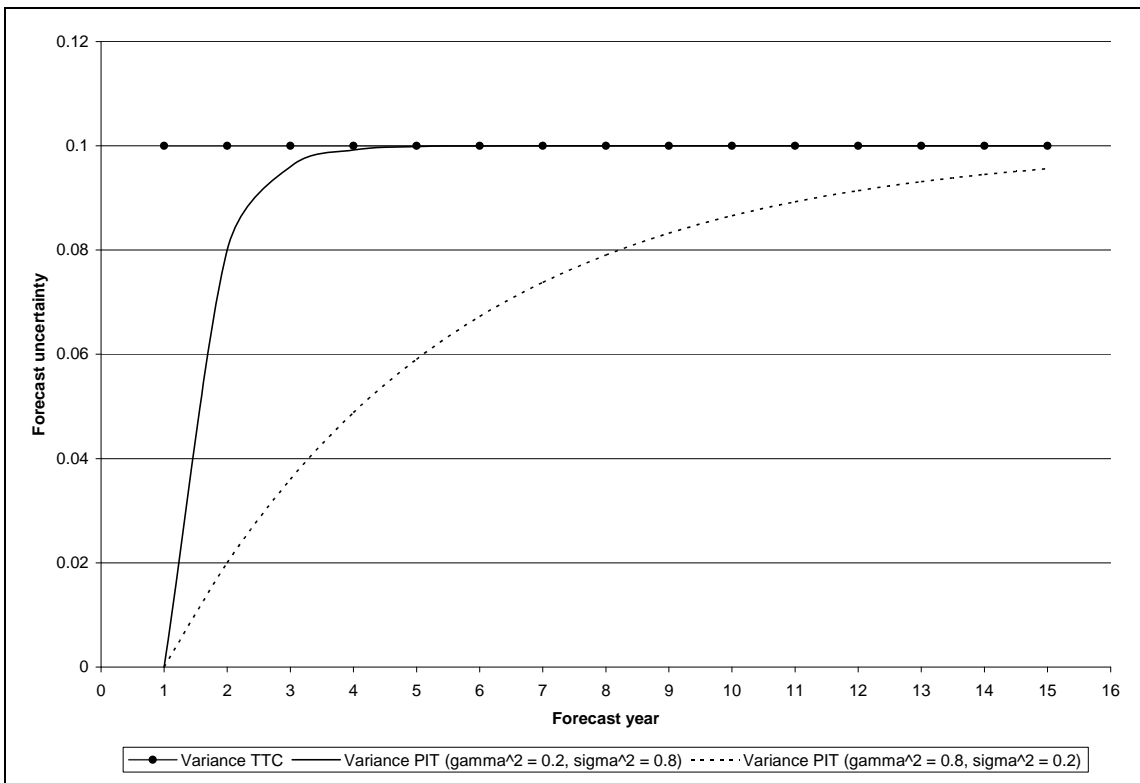
**Figure 1: N.R. - Removed Cumulative Average Default Rates, 1981 - 2005 (%); note that the rating categories 'AAA' and 'AA' are excluded for a better display.**



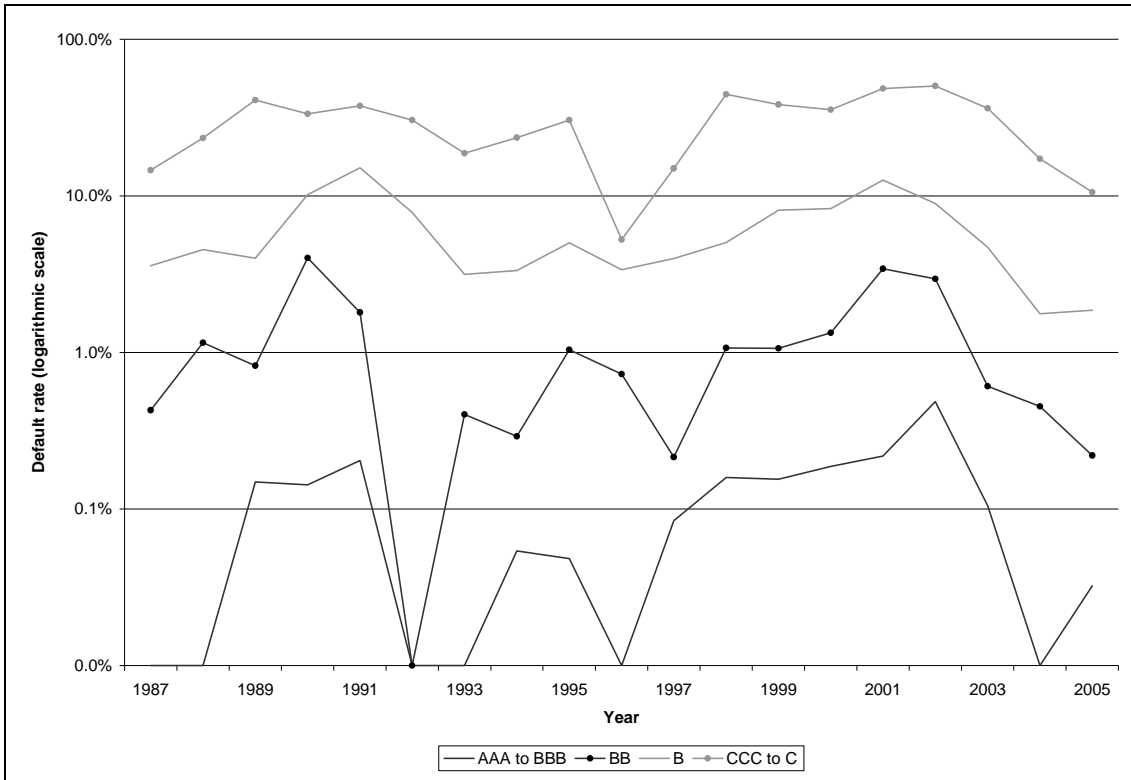
**Figure 2: Variance of the macroeconomic factor in the TTC and PIT model, high business cycle dependency**



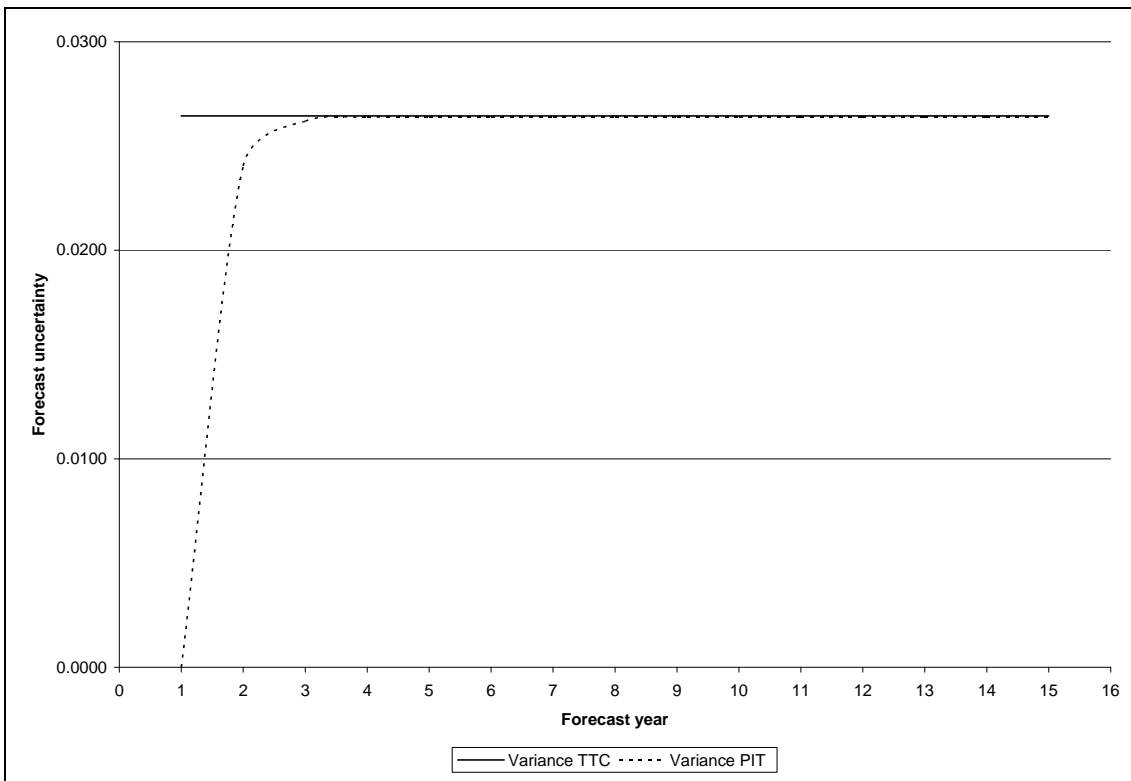
**Figure 3: Variance of the macroeconomic factor in the TTC and PIT model, low business cycle dependency**



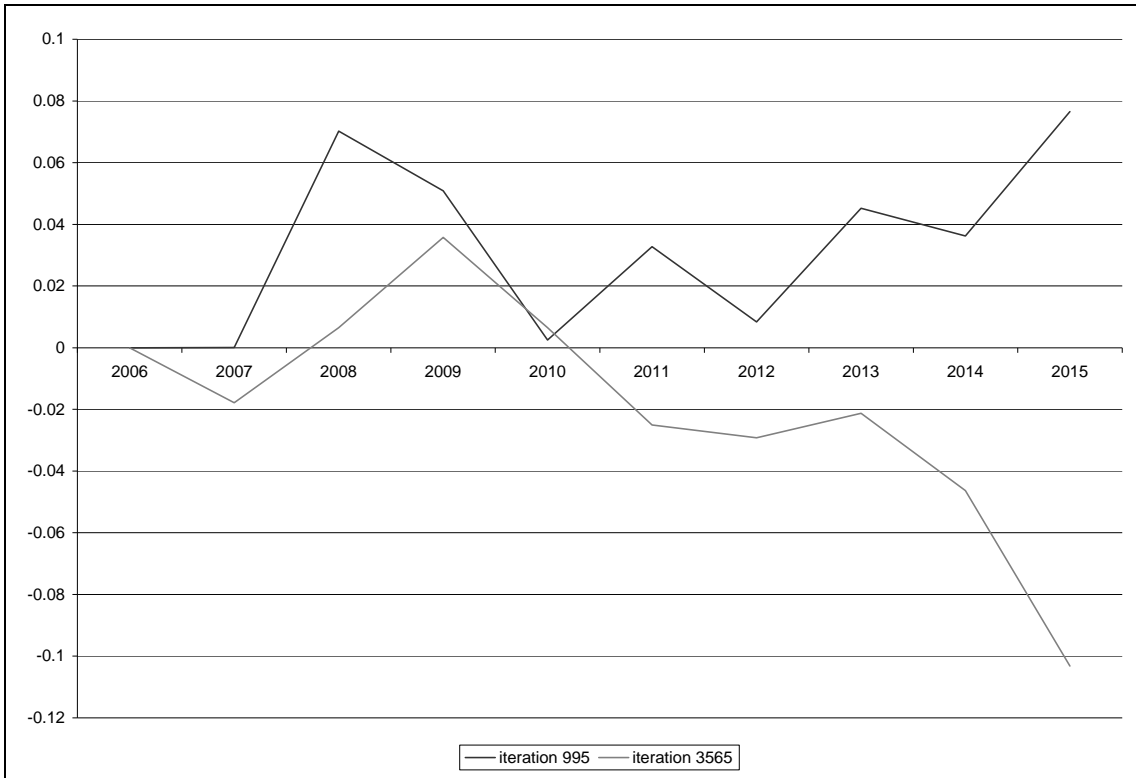
**Figure 4: Historic default rates for different rating categories**



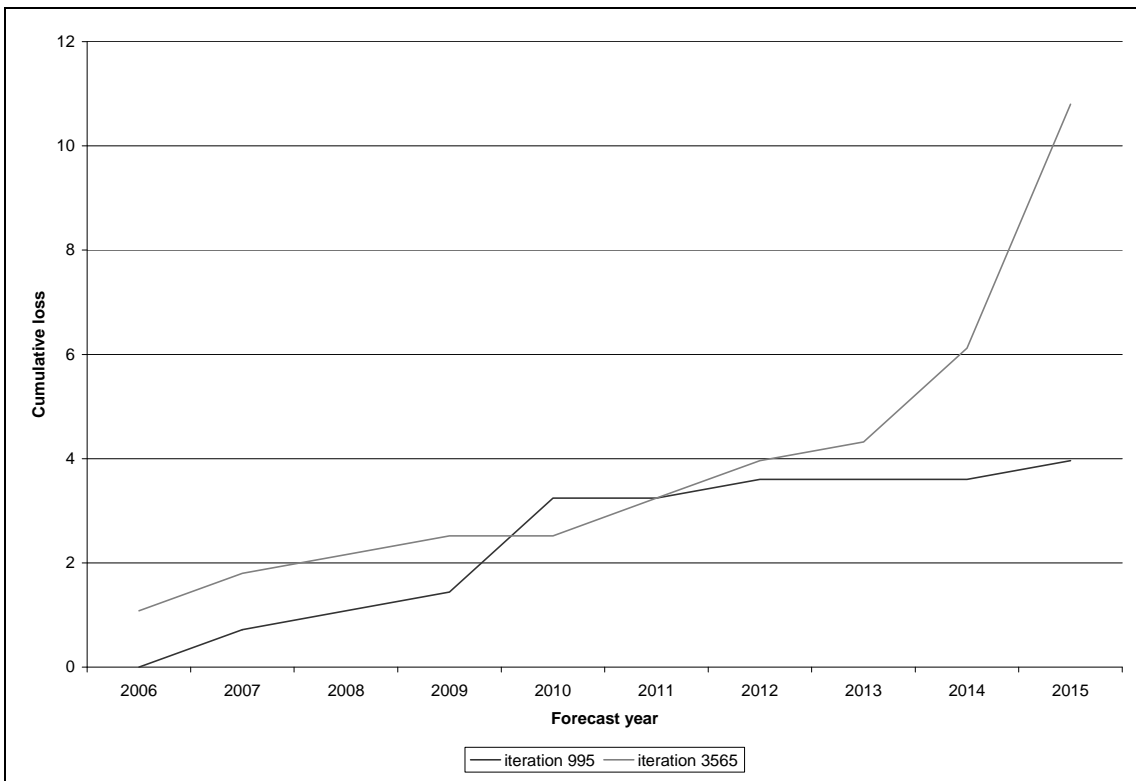
**Figure 5: Variance of the macroeconomic factor in the estimated TTC and PIT model**



**Figure 6: Selected simulated macroeconomic risk drivers**



**Figure 7: Selected cumulative loss distributions**



## Tables

**Table 1: Variance of the macroeconomic risk driver for various periods**

| Period | Variance of macroeconomic risk driver        |
|--------|--|
| T+1    | 0  |
| T+2    | $\beta^2 \sigma^2$                           |
| T+3    | $\beta^2 \sigma^2 (1 + \gamma^2)$            |
| T+4    | $\beta^2 \sigma^2 (1 + \gamma^2 + \gamma^4)$ |

**Table 2: Number of observations and defaults for different rating categories**

| Rating category | Number of defaults | Number of observations |
|-----------------|--------------------|------------------------|
| AAA to BBB      | 48                 | 40908                  |
| BB              | 112                | 9385                   |
| B               | 569                | 9298                   |
| CCC to C        | 378                | 1179                   |
| Total           | 1107               | 60770                  |

**Table 3: Number of observations and defaults for different years**

| Year  | Number of defaults | Number of observations |
|-------|--------------------|------------------------|
| 1987  | 19                 | 1833                   |
| 1988  | 30                 | 1936                   |
| 1989  | 36                 | 1981                   |
| 1990  | 58                 | 2001                   |
| 1991  | 68                 | 2013                   |
| 1992  | 30                 | 2095                   |
| 1993  | 13                 | 2181                   |
| 1994  | 16                 | 2509                   |
| 1995  | 30                 | 2842                   |
| 1996  | 16                 | 3004                   |
| 1997  | 22                 | 3264                   |
| 1998  | 53                 | 3719                   |
| 1999  | 96                 | 4066                   |
| 2000  | 109                | 4313                   |
| 2001  | 179                | 4396                   |
| 2002  | 172                | 4465                   |
| 2003  | 93                 | 4479                   |
| 2004  | 37                 | 4708                   |
| 2005  | 30                 | 4965                   |
| Total | 1107               | 60770                  |

**Table 4: Descriptive statistics for the variables of the data set**

|                         | Mean   | Median | Maximum | Minimum | Std. Dev. | Skewness | Kurtosis |
|-------------------------|--------|--------|---------|---------|-----------|----------|----------|
| Default rate AAA to BBB | 0.0011 | 0.0008 | 0.0048  | 0.0000  | 0.0012    | 1.6253   | 6.0864   |
| Default rate BB         | 0.0116 | 0.0082 | 0.0402  | 0.0000  | 0.0113    | 1.4032   | 3.8770   |
| Default rate B          | 0.0607 | 0.0470 | 0.1509  | 0.0176  | 0.0367    | 1.0062   | 3.1712   |
| Default rate CCC to C   | 0.2918 | 0.3043 | 0.5034  | 0.0526  | 0.1313    | -0.1352  | 1.9647   |
| LEAD                    | 0.0000 | 0.0095 | 0.0457  | -0.0700 | 0.0301    | -0.8568  | 3.1573   |

**Table 5: Bravais-Pearson correlation coefficients for the variables of the data set**

|                         | Default rate AAA to BBB | BB     | B      | CCC to C | LEAD    |
|-------------------------|-------------------------|--------|--------|----------|---------|
| Default rate AAA to BBB | 1                       | 0.6666 | 0.6097 | 0.7830   | -0.4398 |
| Default rate BB         |                         | 1      | 0.7202 | 0.6034   | -0.7200 |
| Default rate B          |                         |        | 1      | 0.6666   | -0.8178 |
| Default rate CCC to C   |                         |        |        | 1        | -0.3689 |
| LEAD                    |                         |        |        |          | 1       |

**Table 6: Estimated Parameters for various models**

Annual default data from 1987 to 2005; parameter estimates for the PIT and TTC model; standard errors are in parentheses; \*\*\* significant at 1% level, \*\* significant at 5% level, \* significant at 10% level.

|              | PIT                    |                        |                       | TTC                    |                       |
|--------------|------------------------|------------------------|-----------------------|------------------------|-----------------------|
| Rating class | $\alpha$               | $\beta$                | $w$                   | $\alpha$               | $\nu$                 |
| AAA to BBB   | -3.0864***<br>(0.0708) | -5.1647**<br>(2.3489)  | 0.2076***<br>(0.0666) | -3.0594***<br>(0.0792) | 0.2484***<br>(0.0720) |
| BB           | -2.3181***<br>(0.0582) | -8.1524***<br>(2.0416) | 0.1478***<br>(0.0474) | -2.2712***<br>(0.0817) | 0.2825***<br>(0.0597) |
| B            | -1.5876***<br>(0.0426) | -7.7506***<br>(1.4431) | 0.1535***<br>(0.0341) | -1.5500***<br>(0.0674) | 0.2661***<br>(0.0453) |
| CCC to C     | -0.5322***<br>(0.0776) | -5.4031*<br>(2.5971)   | 0.2781***<br>(0.0625) | -0.5279***<br>(0.0872) | 0.3271***<br>(0.0631) |

**Table 7: Estimated asset correlation for the PIT (various years) and TTC model.**

|              | 2006    | 2007    | 2008    | All future years |
|--------------|---------|---------|---------|------------------|
| Rating class | PIT     | PIT     | PIT     | TTC              |
| AAA to BBB   | 0.04309 | 0.06366 | 0.06545 | 0.06172          |
| BB           | 0.02186 | 0.07263 | 0.07690 | 0.07980          |
| B            | 0.02357 | 0.06961 | 0.07351 | 0.07081          |
| CCC to C     | 0.07734 | 0.09900 | 0.1009  | 0.1070           |

**Table 8: Characteristics of the forecast loss distributions for different models; maturity of credit portfolio equals 1 year**

| Rating class | Model | Expected loss (EL) | Median | 95 <sup>th</sup> percentile | 99 <sup>th</sup> p. | 99.9 <sup>th</sup> p. | 99.9 <sup>th</sup> CVaR |
|--------------|-------|--------------------|--------|-----------------------------|---------------------|-----------------------|-------------------------|
| AAA to BBB   | PIT-1 | 0.05               | 0.00   | 0.36                        | 0.72                | 0.72                  | 0.67                    |
|              | PIT-2 | 0.06               | 0.00   | 0.36                        | 0.72                | 1.08                  | 1.03                    |
|              | TTC   | 0.05               | 0.00   | 0.36                        | 0.72                | 1.08                  | 1.03                    |
| BB           | PIT-1 | 0.46               | 0.36   | 1.44                        | 1.80                | 2.52                  | 2.06                    |
|              | PIT-2 | 0.58               | 0.36   | 1.44                        | 2.16                | 2.88                  | 2.30                    |
|              | TTC   | 0.52               | 0.36   | 1.80                        | 2.52                | 3.96                  | 3.44                    |
| B            | PIT-1 | 2.53               | 2.52   | 4.68                        | 6.12                | 7.56                  | 5.03                    |
|              | PIT-2 | 2.99               | 2.88   | 5.40                        | 6.84                | 8.28                  | 5.29                    |
|              | TTC   | 2.72               | 2.52   | 6.12                        | 8.28                | 11.16                 | 8.44                    |
| CCC to C     | PIT-1 | 13.36              | 12.96  | 21.60                       | 25.56               | 29.70                 | 16.35                   |
|              | PIT-2 | 14.30              | 14.04  | 22.68                       | 26.64               | 30.78                 | 16.48                   |
|              | TTC   | 13.42              | 12.96  | 23.04                       | 27.72               | 32.04                 | 18.62                   |

**Table 9: Characteristics of the forecast loss distributions for different models; maturity of credit portfolio equals 10 years**

| Rating class | Model | Expected loss (EL) | Median | 95 <sup>th</sup> percentile | 99 <sup>th</sup> p. | 99.9 <sup>th</sup> p. | 99.9 <sup>th</sup> CVaR |
|--------------|-------|--------------------|--------|-----------------------------|---------------------|-----------------------|-------------------------|
| AAA to BBB   | PIT-1 | 0.51               | 0.36   | 1.44                        | 1.80                | 2.52                  | 2.01                    |
|              | PIT-2 | 0.52               | 0.36   | 1.44                        | 1.80                | 2.52                  | 2.00                    |
|              | TTC   | 0.50               | 0.36   | 1.44                        | 1.80                | 2.52                  | 2.02                    |
| BB           | PIT-1 | 5.10               | 5.04   | 8.28                        | 9.72                | 11.52                 | 6.42                    |
|              | PIT-2 | 5.26               | 5.04   | 8.28                        | 10.08               | 11.88                 | 6.62                    |
|              | TTC   | 4.94               | 4.68   | 7.92                        | 9.72                | 11.52                 | 6.58                    |
| B            | PIT-1 | 20.88              | 20.88  | 26.64                       | 28.80               | 31.32                 | 10.45                   |
|              | PIT-2 | 21.26              | 21.24  | 26.64                       | 29.16               | 31.68                 | 10.42                   |
|              | TTC   | 20.89              | 20.88  | 26.64                       | 28.80               | 31.32                 | 10.43                   |
| CCC to C     | PIT-1 | 43.71              | 43.92  | 45.00                       | 45.00               | 45.00                 | 1.29                    |
|              | PIT-2 | 43.77              | 43.92  | 45.00                       | 45.00               | 45.00                 | 1.23                    |
|              | TTC   | 43.70              | 43.92  | 45.00                       | 45.00               | 45.00                 | 1.30                    |

**Table 10: Regulatory and economic capital; maturity of credit portfolio equals 1 year**

| Rating class | Model | Regulatory capital | 99.9 <sup>th</sup> CVaR |
|--------------|-------|--------------------|-------------------------|
| AAA to BBB   | PIT-1 | 1.89               | 0.67                    |
|              | PIT-2 | 2.15               | 1.03                    |
|              | TTC   | 2.01               | 1.03                    |
| BB           | PIT-1 | 7.39               | 2.06                    |
|              | PIT-2 | 8.15               | 2.30                    |
|              | TTC   | 7.78               | 3.44                    |
| B            | PIT-1 | 13.80              | 5.03                    |
|              | PIT-2 | 14.80              | 5.29                    |
|              | TTC   | 14.22              | 8.44                    |
| CCC to C     | PIT-1 | 23.60              | 16.35                   |
|              | PIT-2 | 23.61              | 16.48                   |
|              | TTC   | 23.61              | 18.62                   |

**Table 11: Characteristics of the forecast loss distributions for different investment tranches**

| Tranche  | Attachment point | Detachment point | EL PIT-1 | EL TTC | EL PIT-1 | EL TTC   |
|----------|------------------|------------------|----------|--------|----------|----------|
|          |                  |                  | 1 year   | 1 year | 10 years | 10 years |
| 1        | 0                | 3                | 0.4590   | 0.5143 | 2.9384   | 2.9204   |
| 2        | 3                | 7                | 0.0000   | 0.0037 | 2.0045   | 1.8815   |
| 3        | 7                | 10               | 0.0000   | 0.0000 | 0.1456   | 0.1303   |
| 4        | 10               | 15               | 0.0000   | 0.0000 | 0.0068   | 0.0057   |
| 5        | 15               | 30               | 0.0000   | 0.0000 | 0.0000   | 0.0000   |
| 6        | 30               | 100              | 0.0000   | 0.0000 | 0.0000   | 0.0000   |
| $\Sigma$ |                  |                  | 0.4591   | 0.5180 | 5.0953   | 4.9378   |