Understanding the risks in and rewards for pairs-trading

Michael T. Chng*

Current draft 25th August, 2007

Abstract

Contrarian and momentum trading have both received ample academic attention over the past two decades. Ironically, pairs-trading, which has been around Wall Street for over twenty years and is also based on past return dynamics, has been largely ignored. We show that while the expected profit which stems from a portfolio of pairwise stocks contain both contrarian and momentum profit components, there exists a profit source that is unique to pairs-trading. We discuss two related applications of our model. First, it can be used to empirically measure and contrast the economic significance of various profit sources. Second, it can then be used to analyze the sensitivity of various profit sources to the number and type of matching restrictions. In doing so, we offer a better understanding of the risks in and rewards for pairs-trading.

JEL classification: G14, G15.

Key words and phrases: common factors, negative serial covariance, trading strategy.

*Corresponding author's contacts: Department of Finance, University of Melbourne, VIC 3010, Australia. Tel: 613-83447660. Email: mchng@unimelb.edu.au. Funding from the Melbourne Center for Financial Studies is gratefully acknowledged. I retain full property rights to all existing errors.

Understanding the risks in and rewards for pairs-trading

1 Introduction

Pairs-trading is widely perceived as a form of technical analysis used by hedge funds and boutique investment houses. The process involves exhaustively matching and ranking pairwise stocks (i, j) based on some pre-specified measure of 'closeness'. This is to identify firms whose historical prices move closely together over time. After matched stocks are identified, the price gap $s_{nt} = p_{it} - p_{jt}$ between component stocks i and j of a given pair $n, n = 1, 2, ..., N$ acts as a signal to open and close an often near market-neutral position. If s_{nt} exceeds a pre-specified threshold ε, a long-short position is created by short-selling the dearer stock to fund the purchase of the cheaper stock. Patterns in s_{nt} over time e.g negative serial covariance $\sigma_{s_{nt},s_{nt-1}} < 0$ implies positive expected profit from holding a zero-cost N-pair portfolio.

There are several reasons for the popularity of pairs-trading. First, the procedure is simple to understand and execute. Second, valuation models, which are subjected to wide error margins, are not required since pairs-trading is based on relative valuation. Third, it is sufficiently flexible to accommodate various investment styles. Lastly, it normally does not evoke frequent intraday re-balancing, such that actual trading can be automated and is feasibly profitable. Despite its history and popularity, the nature of pairs-trading remains elusive. It is commonly branded as statistical arbitrage, and yet pairs-trading relates to 'near' law-of-one-price, relative valuation, contrarian principles and cointegration. While its basic idea and procedures are straightforward, pairs-trading can be complicated by the calibration of a set of parameters. These include i) the matching criterion to identify pairwise stocks ii) whether to match across the population of stocks i.e unrestricted matching, or to within or across stratified samples i.e restricted matching iii) the type and number of restrictions iv) the threshold values for opening and closing positions v) pairs-portfolio size. In practice, these parameters are often determined in an ad-hoc fashion.

To the best of our knowledge, Elliott et al (2005) and Gatev, Goetzmann and Rouwenhorst, henceforth GGR (2006), are the only two recent finance articles on pairs-trading. A structured framework to calibrate the various parameters of pairs-trading would no doubt attract practitioners' attention. Although that task is currently too complex, a necessary first step will require some understanding on the nature of pairs-trading. What are the sources of its rewards? What are the risks involved? How are the profit sources affected by the choice of parameters e.g. the types and/or number of restrictions. Price formation models, a cornerstone of the market microstructure literature, are the result of academic endeavors (Glosten and Milgrom (1985); Easley and O'Hara (1987); Brown and Jennings (1989); Hasbrouck (1991, 1993, 1995) and Wang (1994)) to turn technical analysis from an art to a science.

We have two objectives in this paper. First, we derive an analytical expected profit function $\mathbb{E}(\Pi)$ for a portfolio of pairwise stocks to identify various profit components. This facilitates an empirical measure of their economic significance, which then allows us to establish how the calibration of certain parameters could actually improve pairs-trading profitability.

E.g. Finding strong evidence of momentum in idiosyncratic returns, Grundy and Martin (2001) show that momentum strategies that form winner and loser portfolios based on idiosyncratic returns outperform comparable strategies based on total returns. This constitutes a calibration of the ranking criterion to improve the profitability of momentum trading.

Second, we establish a relationship between pairs-trading profitability and the number of matching restrictions. We show that the $\mathbb{E}(\Pi)$ of pairs-trading based on unrestricted matching is driven by sources different from pairs-trading based on restricted matching. Within the latter case, the profit sources would differ depending on the type and/or number of matching restrictions. Most importantly, our $\mathbb{E}(\Pi)$ gives a better collective appreciation of the voluminous and detailed empirical analysis performed by GGR (2006) to examine the profit robustness of a generic pairs-trading strategy. Achieving both objectives, we offer some insights into the risks in and rewards for pairs-trading.

Our $\mathbb{E}(\Pi)$ reveals that pairs-trading profitability stems from four potential sources. First, it is driven by negative serial covariance in idiosyncratic returns $\sigma_{\varepsilon_{it},\varepsilon_{it-1}} < 0$. This is caused by stock price overreaction to idiosyncratic (firm-specific) news, which is a main source of contrarian profits. Addressing the possibility that pairs-trading is simply contrarian trading in disguise, GGR (2006) perform a bootstrap to contrast the contrarian profits of matched pairs against random pairs. They find that reversion cannot fully explain the profits from matched pairs contrarian trading.

Second, $\mathbb{E}(\Pi)$ contains $(\overline{r}_i - \overline{r}_j)^2$, a squared-discrepancy in the unconditional expected return of pairwise stocks i and j. Since opposite positions are taken in a pair, if $\overline{r}_i$ and $\overline{r}_j$ follow similar factor structures, then the pair would be near market-neutral. This second component allows for the possibility of paired firms displaying dissimilar factor structures. The larger the value of $(\overline{r}_i - \overline{r}_j)^2$, the greater the difference in common factor exposures. Since such a long-short position in (i, j) is not market neutral, the net factor exposure(s) requires a higher $\mathbb{E}(\Pi)$. GGR (2006) address this issue with a sub-sample analysis to contrast the results between the top ranked pairs portfolio 1-20 against the bottom ranked pairs portfolio 101-120. This analysis they claim is "valuable because most of the top pairs share certain characteristics...".

Third, GGR (2006) consider five risk factors to examine the robustness of pairs-trading profit. In addition to the Fama and French (1996) β_m, HML and SMB factors, GGR (2006) also include a momentum factor and a reversal factor to adjust for dynamics in price response by component stocks to common factors. The return generating process specified in our model follows a K-factor structure, which allows us to incorporate the Fama-French factors. For each factor, we allow stock returns exposure to both a contemporaneous and a lagged factor innovation. By doing so, we are able to incorporate dynamics in price response by component stocks to unexpected factor realizations. In our $\mathbb{E}(\Pi)$, the profit component attributed to common factors encapsulates all 18 possible scenarios pertaining to differences in price responses by (i, j) to unexpected factor realizations. These scenarios can be categorized as delayed reaction, overreaction or a mixture of both. This profit component facilitates a direct measure of the contribution to $\mathbb{E}(\Pi)$ by price response dynamics to common factors, which directly address the same issue that GGR (2006) empirically address using momentum and reversal factors.

Fourth, GGR (2006) examine pairs-trading performance based on matching within S&P broad industry groups[1] and find robust profits across industries. In Section 3.2, we explore the impact on $\mathbb{E}(\Pi)$ associated with increasing the number of matching restrictions. We conjecture that when firms are matched within increasing smaller stratified samples, matched pairs will possess increasingly similar characteristics. We categorize similarity between (i, j) as either competitive[2] or collaborative in nature (e.g. customer/supplier[3]; Firm i is a major shareholder of Firm j). If this conjecture is reasonable, the assumption of zero cross-serial covariance in idiosyncratic returns $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}} = \sigma_{\varepsilon_{jt},\varepsilon_{it-1}} = 0$ made by Jegadeesh and Titman (1995) should be relaxed. The fourth component in $\mathbb{E}(\Pi)$ reveals that pairing up firms that are competitive (collaborative) in nature decreases (increases) pairs-trading profitability.

As $\mathbb{E}(\Pi)$ can be empirically estimated, our model offers a measure of the economic significance of various pairs-trading profit sources. From there, it also allows us to analyze the sensitivity of various profit components when we vary (say) the type and number of matching restrictions.[4] Most importantly, our $\mathbb{E}(\Pi)$ provides an insight into the link between profitability and the calibration of pairs-trading. If the empirical results show (in)significant contribution to $\mathbb{E}(\Pi)$ by the common factor price response component, then pairs-trading based on restricted matching should outperform (under-perform) pairs-trading based on unrestricted matching.

The derivation of $\mathbb{E}(\Pi)$ is outlined in section 2. Empirical applications and hypotheses are discussed in section 3. Section 4 concludes.

2 The sources of pairs-trading expected profit

2.1 Background

Contrarian trading stipulates selling winner stocks and buying loser stocks, and is designed to profit from overreaction and subsequent mean-reversion i.e. negative serial covariance in stock returns. It generally involves ranking stocks based on time $t-1$ returns to take simultaneous short and long positions in the top and bottom decile portfolios at time t. If prices do indeed overreact, then the short-long contrarian position can be closed out at time $t+1$ for a profit. Positive profits are reported in Jegadeesh (1990) and Lehmann (1990). However, Lo and MacKinlay (1990) show that contrarian profits could also be driven by lead-lag effects between winner and loser stocks. If stock j reacts similarly to stock i, but with a delay, then buying (selling) j subsequent to an increase (decrease) in i would be profitable, even though neither stocks overreact. Their results attribute around 50% of contrarian profits to such lead-lag effects. Lo and MacKinlay (1990) show that that negative serial covariance

[1] They are Utilities, Transportation, Financial and Industrial.

[2] E.g Boeing v. Airbus; Citigroup v. HSBC; Google v. Yahoo; Bridgestone-Firestone v. Yokohama Wheels; Dell v. IBM etc

[3] E.g Boeing & Singapore Airline; Motorola & Telstra.

[4] For example, Avramov et al (2006) document a strong (volume-adjusted) relationship between short-run return reversals and illiquidity. We can easily test the sensitivity of various pairs-trading profit components to liquidity-sorted stratified samples.

in returns $\sigma_{r_{it},r_{it-1}} < 0$ and positive cross-serial covariance in returns $\sigma_{r_{it},r_{jt-1}} > 0$ $\forall i \neq j$ are both potential sources of contrarian profits.

Jegadeesh and Titman (1995) note that any lead-lag effects between winner and loser stocks must be due to delayed reaction to common factors. Their analysis of contrarian profits consider overreaction and under-reaction to both common factors and idiosyncratic news. In contrast to Lo and MacKinlay (1990), Jegadeesh and Titman (1995) find that contrarian profits is mainly driven by overreaction to idiosyncratic news. This is consistent with their observation that while systematic overreaction to idiosyncratic news will generate contrarian profits, systematic overreaction to common factors may actually dilute contrarian profits. The main extension by Jegadeesh and Titman (1995) to the Lo and MacKinlay (1990) analysis is to demonstrate common factor price reaction as a more appropriate measure of contrarian profits attributable to lead-lag effects compared to $\sigma_{r_{it},r_{jt-1}}$ and $\sigma_{r_{jt},r_{it-1}}$.

We apply the Lo and MacKinlay portfolio weighting scheme on the Jegadeesh and Titman (1995) factor model based profit decomposition to identify various components of $\mathbb{E}(\Pi)$ from a portfolio of pairwise stocks. Jegadeesh and Titman (1995) assume zero cross-serial covariance in idiosyncratic returns, such that lead-lag effects is driven entirely by price response to common factors. We explain in Section 3 that if the type and number of matching restrictions resulted in pairwise firms sharing increasingly idiosyncratic relationships that are either competitive or collaborative in nature, then non-zero cross-serial covariance in idiosyncratic returns constitutes another source of pairs-trading profitability.

2.2 The nature of pairs-trading

Contrarian or momentum trading involves ranking stocks based on past returns during the formation horizon $(T - t)$ to identify winner and loser portfolios. In pairs-trading, stocks are ranked according to how close they move together over time based on some pre-specified measure of closeness.[5] Conceptually, what pairs-trading does is to create and trade in a set of N synthetic securities[6] with prices $\{s_{1t}, s_{2t}, ..., s_{Nt}\}$ and $\mathbb{E}(s_n) = 0$ $\forall n$.[7] Unlike the price of a tradable security, s_{nt} can be negative since s_{nt} is denoted as $p_{it} - p_{jt}$. If $s_{nt} > (<) + (-)\varepsilon$, sell (buy) pair n at s_{nt}, which is equivalent to short-selling (buying) stock i and buying (short-selling) stock j.[8] There is a natural conceptual link between pairs-trading and cointegration/error-correction between non-stationary time-series proposed by Engle and Granger (1987).

[5]Examples of closeness critera include minimum Euclidean distance, maximum correlation coefficient, lowest p-value for cointegration test statistics.

[6]When all possible pairwise combinations are considered, the number of synthetic securities outstrips the corresponding actual number of securities. For every N securities in a market, there are $\{p_{1t}, ...p_{Nt}\}$ prices and $\frac{N!}{2(N-2)!}$ price gaps. E.g. A stock market that contains 100 stocks has 100 price series and 100!/2*98!=4,950 price gap series. This implies more opportunities from trading pairs of securities than the securities themselves in (say) a valuation context.

[7]To note, pairs-trading only requires $\mathbb{E}(s_n) = c$. Without loss of generality, we consider the simple case of $c = 0$.

[8]This perception is similar to that of a fixed-for-floating interest rate swap, where the seller of the swap is simultaneously selling the floating rate bond and buying the fixed rate bond. As such, the price of a swap is simply the price differential between its fixed and floating components.

Pairs-trading requires calibration of an array of parameters listed below. In practise, these parameters are often determined in an ad-hoc fashion.[9] During formation, the population of listed firms is screened to remove any stocks that do not trade for one or more days. This removes from consideration illiquid stocks that hinders the actual trading of matched pairs. It also removes the moot outcome of creating a pair from two stocks whose initial prices are close and they remain static during $(T-t)$.

1. A pre-specified (set of) matching criterion to identify and rank pairwise stocks
2. Unrestricted versus restricted matching
3. The type and number of matching restrictions
4. The threshold values for opening and closing positions
5. Pairs-portfolio size N

The closeness measure used in GGR (2006) is to minimize the Euclidian distance between the prices of exhaustively matched firms, normalized for reinvested dividends and number of shares outstanding $\mathrm{Min}\sqrt{\sum_t^T (p_{it} - p_{jt})^2}$. This minimum distance criterion corresponds to the notion of pairing up stocks whose prices historically move closely together. At the end of $(T-t)$, the measures are ranked to identify pairs $n = 1, 2, ..., N$ of similar stocks (i, j) for subsequent consideration in the trading horizon $(U-T)$, which commences the next trading day onwards.[10] Unlike contrarian or momentum trading, positions in short-listed pairwise stocks do not open at the onset of the trading horizon. Instead, trading in a given pair n during $(U-t)$ is triggered when s_{nt} exceeds a pre-specified threshold value ε. There is no paradigm for setting optional threshold value(s). GGR (2006) set the opening trigger as two historical standard deviations $|s_{nT_1}| > \varepsilon = 2\sqrt{\sum_t^T s_{nt}^2}$ to be consistent with their matching criterion. An existing position in a given pair is closed only when $s_{nT_2} = 0$.[11] If prices of opened pairs do not converge by the end of $(U-T)$, gains/losses will be calculated based on closing prices on the last trading day. In sum, a given pair n can be described by one of the following situations during $(U-T)$:

1. No trade whatsoever
2. A position is opened, but is closed only by default at time U
3. A position is opened & closed once i.e. a complete cycle, then either 1) or 2)
4. Similar to 3), but for two or more complete cycles

[9] To reiterate, the objective of our paper is to identify the various sources of pairs-trading profits. We address questions relating to the optimal setting of pairs-trading parameters in future research.

[10] It is possible for the same stock to appear in two or more matched pairs that qualify for consideration during $(U-T)$. For simplicity, we can impose the constraint that a stock can only be involved in one pair. Subsequent empirical analysis could examine how the repeat appearance of a given stock affects profitability.

[11] As such, it is possible for a given pair n to trade more than once during $(U-T)$.

2.3 Decomposition of pairs-trading profits

$$
\begin{aligned}
r_{it} &= \overline{r}_i + \sum_{k=1}^{K}(b_{0ik}f_{kt} + b_{1ik}f_{kt-1}) + \varepsilon_{it} \\
r_{jt} &= \overline{r}_j + \sum_{k=1}^{K}(b_{0jk}f_{kt} + b_{1jk}f_{kt-1}) + \varepsilon_{jt}
\end{aligned} \tag{1}
$$

Denote $r_{it} = p_{it} - p_{it-1}, s_{nt} = p_{it} - p_{jt}, \Delta s_{nt} = s_{nt} - s_{nt-1} = r_{it} - r_{jt}$ and $\Delta\overline{s}_t = \frac{1}{N}\sum_{n=1}^{N}\Delta s_{nt}$. Consider the following K-factor return generating processes of stock i and j respectively. The terms $\overline{r}_i$ and $\overline{r}_j$ are the unconditional expected returns of stock i and j respectively; f_{kt} is the unexpected k^{th}-factor realization i.e $\sigma_{f_{kt}f_{kt-1}} = 0$. The coefficients b_{0ik} and b_{1ik} are the sensitivities of stock i to the contemporaneous and lagged factor realizations. Lastly, ε_{it} and ε_{jt} are the corresponding idiosyncratic returns on stock i and stock j.

$$
\begin{aligned}
\mathbb{E}[\Delta s_{nt} - \Delta\overline{s}_t | \Delta s_{nt-1} - \Delta\overline{s}_{t-1} > 0] < 0 \\
\mathbb{E}[\Delta s_{nt} - \Delta\overline{s}_t | \Delta s_{nt-1} - \Delta\overline{s}_{t-1} < 0] > 0 \\
\therefore -\mathbb{E}[(\Delta s_{nt} - \Delta\overline{s}_t)(\Delta s_{nt-1} - \Delta\overline{s}_{t-1})] > 0
\end{aligned} \tag{2}
$$

The persistence of pairs-trading profitability implies the condition specified in equation (2), which states that pairwise stocks which experience above average widening in price gaps at time $t-1$ relative to their peers are expected to have their gaps narrowed at time t. We adopt the Lehmann (1990) and Lo and MacKinlay (1990) portfolio weighting scheme in our paper because it is analytically tractable and it allows our $\mathbb{E}(\Pi)$ to encompass equation(2).

$$
w_{nt} = -\frac{1}{N}(\Delta s_{nt-1} - \Delta\overline{s}_{t-1}) \tag{3}
$$

For a N-pair portfolio, the weight w_{nt} assign to pair n at time t is defined in equation (3). It takes into account the difference between the change in the gap Δs_{nt-1} and its cross-sectional mean $\Delta\overline{s}_{t-1}$ at time $t-1$. If s_{nt} exhibits a positive (negative) widening, the weight assigned to pair n would be a large negative (positive) value to indicate a short-sell (purchase) i.e. short-sell (buy) stock i and buy (short-sell) stock j. While w_{nt} is not necessarily zero, equation (4) shows that the total investment in a N-pair portfolio is zero by construction.

$$
\sum_{n=1}^{N} w_{nt} = -\frac{1}{N}[\sum_{n=1}^{N}\Delta s_{nt-1} - N\Delta\overline{s}_{t-1}] = 0 \tag{4}
$$

Equation (5) outlines the N-pair portfolio profit Π_t at time t, where Π_t is the cross-sectional weighted average change in price gap. If $\Delta s_{nt-1} < 0$, then $w_{nt} > 0$, and if $\Delta s_{nt} > 0$, Π_t increases. In words, a negative change in the price gap for pair n at time $t-1$ triggers a purchase. This is subsequently closed out at a profit when pair n exhibits a positive change i.e. the price gap narrows at time t, and vice versa.

$$
\begin{aligned}
\Pi_t = \sum_{n=1}^{N} w_{nt}\Delta s_{nt} &= -\frac{1}{N}[\sum_{n=1}^{N}(\Delta s_{nt-1} - \Delta\overline{s}_{t-1})\Delta s_{nt}] \\
&= -\frac{1}{N}[\sum_{n=1}^{N}(\Delta s_{nt}\Delta s_{nt-1}) - N\Delta\overline{s}_{t-1}\sum_{n=1}^{N}\Delta s_{nt}] \\
&= \frac{-\sum_{n=1}^{N}(\Delta s_{nt}\Delta s_{nt-1}) + \sum_{n=1}^{N}\Delta s_{nt}\sum_{n=1}^{N}\Delta s_{nt-1}}{N}
\end{aligned}
\tag{5}
$$

Equation (5) can be used to derive a general expression for the expected pairs-trading profit function $\mathbb{E}(\Pi)$ in equation (6). It shows that the profitability of a N-pair portfolio is driven by the first-order serial-cross serial covariance matrices of pairwise stocks. It also depends on the squared difference between $\overline{r}_i$ and $\overline{r}_j$. Intuitively, a difference in the unconditional expected return implies dissimilar factor exposures between stock i and stock j. As such, taking opposite positions in i and j does not lead to a market neutral pair. This net factor exposure requires a higher $\mathbb{E}(\Pi)$.

$$
\begin{aligned}
\mathbb{E}(\Pi) &= -\frac{\sum_{n=1}^{N}}{N}[\mathbb{E}(\Delta s_{nt}\Delta s_{nt-1}) - \mathbb{E}(\Delta s_{nt})\mathbb{E}(\Delta s_{nt-1})] \\
&= -\frac{\sum_{n=1}^{N}}{N}[\sigma_{\Delta s_{nt},\Delta s_{nt-1}}] \\
&= -\frac{\sum_{n=ij}^{N}}{N}[\begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}\sigma_{r_{it},r_{it-1}} & \sigma_{r_{jt},r_{it-1}} \\ \sigma_{r_{it},r_{jt-1}} & \sigma_{r_{jt},r_{jt-1}}\end{pmatrix}\begin{pmatrix} & 1 \\ - & 1\end{pmatrix} - (\overline{r}_i - \overline{r}_j)^2]
\end{aligned}
\tag{6}
$$

In order to gain more insight into the various sources of pairs-trading profitability, we further decompose the various serial and cross-serial covariances terms. First, consider the serial covariance in returns on stock i $\sigma_{r_{it},r_{it-1}} = \mathbb{E}(r_{it}r_{it-1}) - \overline{r}_i^2$. Substituting the return generating processes in equation (1) into $\sigma_{r_{it},r_{it-1}}$, we yield the following:

$$\sigma_{r_{it},r_{it-1}} = \mathbb{E}(r_{it}r_{it-1}) - \overline{r}_i^2$$
$$= \mathbb{E}(\overline{r}_i^2 + \overline{r}_i \sum_{k=1}^{K}(b_{0ik}f_{kt-1} + b_{1ik}f_{kt-2}) + \overline{r}_i\varepsilon_{it-1}$$
$$+ \overline{r}_i \sum_{k=1}^{K}(b_{0ik}f_{kt} + b_{1ik}f_{kt-1}) + \sum_{k=1}^{K}(b_{0ik}f_{kt} + b_{1ik}f_{kt-1}) \sum_{k=1}^{K}(b_{0ik}f_{kt-1} + b_{1ik}f_{kt-2})$$
$$+ \varepsilon_{it-1} \sum_{k=1}^{K}(b_{0ik}f_{kt} + b_{1ik}f_{kt-1}) + \overline{r}_i\varepsilon_{it} + \varepsilon_{it} \sum_{k=1}^{K}(b_{0ik}f_{kt-1} + b_{1ik}f_{kt-2}) + \varepsilon_{it}\varepsilon_{it-1}) - \overline{r}_i^2$$

Assume factors are orthogonal, such that $\sigma_{f_{lt}f_{mt}} = 0 \ {\scriptstyle \forall l \neq m}$. And since f_{kt} is an unexpected factor realization, $\sigma_{f_{lt},f_{mt-1}} = 0 \ {\scriptstyle \forall l \neq m}$ and $\mathbb{E}(f_{kt}^2) = \sigma_{f_k}^2$. Also, assume $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}} = 0 \ {\scriptstyle \forall i \neq j}$ i.e no cross-serial covariances in idiosyncratic returns. Lastly, by definition, $\mathbb{E}(f_k) = \mathbb{E}(\overline{r}_i f_k) = \mathbb{E}(\overline{r}_i\varepsilon_{it}) = \mathbb{E}(f_{kt}\varepsilon_{it}) = 0$. Under these assumptions, $\sigma_{r_{it},r_{it-1}}$ collapses to equation (7), which shows that serial covariance in the return on stock i is driven by the dynamics of expected price responses $\mathbb{E}(b_{0ik}b_{1ik})$ to common factors σ_{fk}^2 as well as serial covariance in idiosyncratic returns $\sigma_{\varepsilon_{it}\varepsilon_{it-1}}$. If the price of stock i is expected to fully respond to $\sigma_{fk}^2 \forall k$ at either time t or $t-1$, then either $\mathbb{E}(b_{1ik})$ or $\mathbb{E}(b_{0ik}) = 0$, such that $\mathbb{E}(b_{0ik}b_{1ik}) = 0$. In addition, if idiosyncratic returns are serially uncorrelated i.e. $\sigma_{\varepsilon_{it}\varepsilon_{it-1}} = 0$, then $\sigma_{r_{it},r_{it-1}} = 0$.

$$\sigma_{r_{it},r_{it-1}} = \mathbb{E}[\sum_{k=1}^{K}(b_{0ik}b_{1ik}f_{kt-1}^2)] + \varepsilon_{it}\varepsilon_{it-1}) = \sum_{k=1}^{K}\mathbb{E}(b_{0ik}b_{1ik})\sigma_{f_k}^2 + \sigma_{\varepsilon_{it}\varepsilon_{it-1}} \tag{7}$$

To note, while $\sigma_{r_{jt},r_{jt-1}}$ can be similarly obtained, the off-diagonal cross serial covariances $\sigma_{r_{it},r_{jt-1}}$ and $\sigma_{r_{jt},r_{it-1}}$ are slightly different. If we assume that idiosyncratic returns are cross-serially uncorrelated $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}} = \sigma_{\varepsilon_{jt},e_{jt-1}=0}$, then if $\sigma_{r_{it},r_{jt-1}} \neq 0$, it can only be induced by stock j leading stock i in response to σ_{fk}^2. This is measured by $\mathbb{E}(b_{0jk}b_{1ik})$. Conversely, if stock i responds to σ_{fk}^2 ahead of stock j, then $\sigma_{r_{jt},r_{it-1}} \neq 0$, which is measured by $\mathbb{E}(b_{0ik}b_{1jk})$. If neither stock is expected to react with a delay, $\mathbb{E}(b_{1ik}) = \mathbb{E}(b_{1jk}) = 0$. If both stocks are expected to react with a delay, $\mathbb{E}(b_{0ik}) = \mathbb{E}(b_{0jk}) = 0$. In both cases, $\sigma_{r_{it},r_{jt-1}} = \sigma_{r_{jt},r_{it-1}} = 0$ i.e. no cross-serial covariance in returns whatsoever. Lastly, if stocks i and j exhibit dynamics in price response to σ_{fk}^2 that differs across factors, then it is likely for both $\sigma_{r_{jt},r_{it-1}}$ and $\sigma_{r_{it},r_{jt-1}}$ to be non-zero i.e. bi-directional causality. The following summarizes.

$$\begin{pmatrix} \sigma_{r_{it},r_{it-1}} & \sigma_{r_{jt},r_{it-1}} \\ \sigma_{r_{it},r_{jt-1}} & \sigma_{r_{jt},r_{jt-1}} \end{pmatrix} = \sum_{k=1}^{K} \begin{pmatrix} b_{0ik}b_{1ik} & b_{0ik}b_{1jk} \\ b_{0jk}b_{1ik} & b_{0jk}b_{1jk} \end{pmatrix} \sigma_{f_k}^2 + \begin{pmatrix} \sigma_{\varepsilon_{it},\varepsilon it-1} & 0 \\ 0 & \sigma_{\varepsilon_{jt},\varepsilon_{jt-1}} \end{pmatrix}$$
$$= \begin{pmatrix} b_{0i1} & \dots & b_{0iK} \\ b_{0j1} & \dots & b_{0jK} \end{pmatrix} \times \begin{pmatrix} \sigma_{f_1}^2 & 0 & 0\dots & \\ 0 & \sigma_{f_2}^2 & 0\dots & \\ . & . & . & \\ 0 & 0 & 0\dots & \sigma_{f_K}^2 \end{pmatrix} \times \begin{pmatrix} b_{1i1} & b_{1j1} \\ b_{1i2} & b_{1j2} \\ . & . \\ \\ b_{1iK} & b_{1jK} \end{pmatrix} + \begin{pmatrix} \sigma_{\varepsilon_{it},\varepsilon it-1} & 0 \\ 0 & \sigma_{\varepsilon_{jt},\varepsilon_{jt-1}} \end{pmatrix}$$

By substituting the above into equation (6), we obtain a more detailed expression for $\mathbb{E}(\Pi)$, which is outlined in equation (8). It shows that pairs-trading $\mathbb{E}(\Pi)$ comes from one of four potential sources.

First, it is driven by negative serial correlation in the idiosyncratic returns of either or both stocks i and j since $\sigma_{\varepsilon_{it},\varepsilon_{it-1}} < 0$ and/or $\sigma_{\varepsilon_{jt},\varepsilon_{jt-1}} < 0$ increases $\mathbb{E}(\Pi)$. This shows that profitability of pairs-trading is potentially driven by overreaction to firm-specific news. Second $\mathbb{E}(\Pi)$ is driven by a gap between $\overline{r}_i$ and $\overline{r}_j$. To reiterate, pairs are formed based on their historical prices having moved closely together. A non-zero gap in the unconditional expected returns implies a tendency for prices to diverge. Taken together, this translates to negative serial correlation in s_{nt} over time, which pairs-trading profitability depends upon.

$$\begin{aligned}
\mathbb{E}(\Pi) &= -\sum_{n=ij}^{N}[\sum_{k=1}^{K}[\mathbb{E}(b_{0ik}b_{1ik}) + \mathbb{E}(b_{0jk}b_{1jk})]\sigma^2_{f_k} - \sum_{k=1}^{K}[\mathbb{E}(b_{0ik}b_{1jk}) + \mathbb{E}(b_{0jk}b_{1ik})]\sigma^2_{f_k}] \\
&+ \sigma_{\varepsilon_{it},\varepsilon_{it-1}} + \sigma_{\varepsilon_{jt},\varepsilon_{jt-1}} - (\overline{r}_i - \overline{r}_j)^2) \\
&= -\sum_{n=ij}^{N}\sum_{k=1}^{K}(\mathbb{E}(b_{0ik} - b_{0jk})(b_{1ik} - b_{1jk}))\sigma^2_{f_k} - \sum_{n=ij}^{N}((\sigma_{\varepsilon_{it},\varepsilon_{it-1}} + \sigma_{\varepsilon_{jt},\varepsilon_{jt-1}}) - (\overline{r}_i - \overline{r}_j)^2)
\end{aligned} \tag{8}$$

Third, $\mathbb{E}(\Pi)$ depends on the price reaction dynamics of the two component stocks (i, j) to common factors. This is encompassed by the first term of equation (8), which has three components: $\mathbb{E}(b_{0ik} - b_{0jk}), \mathbb{E}(b_{1ik} - b_{1jk})$ and $\sigma^2_{f_k} > 0$. Accordingly, for the timeliness of stock price reaction to common factors to impact positively on $\mathbb{E}(\Pi)$, this requires $\mathbb{E}[(b_{0ik} - b_{0jk})(b_{1ik} - b_{1jk})] < 0$, which in turn requires either $\mathbb{E}(b_{0ik} - b_{0jk}) > 0$ and $\mathbb{E}(b_{1ik} - b_{1jk}) < 0$, or vice versa. $\mathbb{E}(\mathbf{b_0})$ and its mirror image $\mathbb{E}(\mathbf{b_1})$ each lists three pairs of possible scenarios for $\mathbb{E}(b_{0ik} - b_{0jk}) > 0$ and $\mathbb{E}(b_{1ik} - b_{1jk}) < 0$ respectively.

$$\mathbb{E}(\mathbf{b_0}) = \mathbb{E}\begin{pmatrix} (b_{0ik} & - & b_{0jk}) & > 0 \\ ++ & & + & \\ - & & -- & \\ + & & - & \end{pmatrix} \rightarrow \mathbb{E}\begin{pmatrix} (b_{1ik} & - & b_{1jk}) & < 0 \\ + & & ++ & \\ -- & & - & \\ - & & + & \end{pmatrix} = \mathbb{E}(\mathbf{b_1})$$

A few notes on how to read the above expression. First, $-b_{1ik}$ and $+b_{1jk}$ indicate that $b_{1ik} < 0, b_{1jk} > 0$, such that $(b_{1ik} - b_{1jk}) < 0$ regardless of the size of the coefficients. The case $+ + b_{0ik}$ and $+b_{0jk}$ indicate that while both coefficients are positive, $b_{0ik} > b_{0jk}$, such that $(b_{0ik} - b_{0jk}) > 0$. Second, $\mathbb{E}(\mathbf{b_0}) \rightarrow \mathbb{E}(\mathbf{b_1})$ represents nine possible scenarios that generate pairs-trading profit. An example of a delayed reaction scenario is $(++, +) \rightarrow (+, ++)$. Here most of the r_{it} common factor reaction occurs contemporaneously, while stock j reacts primarily to f_{kt-1}. The case $(-, --) \rightarrow (--, -)$ can be described in a similar fashion. The scenarios $(++, +) \rightarrow (--, -)$ and $(--, -) \rightarrow (++, +)$ represent overreaction by both r_{it} and r_{jt} to f_{kt}, with both component stocks subsequently revising to f_{kt-1}. Overreaction by pairwise stocks to a common factor does not imply profitability per se. The widening and subsequent narrowing of s_{nt} occurs if $r_{it}(r_{jt})$ overreacts by more than $r_{jt}(r_{it})$ to f_{kt}, but subsequently also revises by more than $r_{jt}(r_{it})$ to f_{kt-1}.

The converse situation that also contributes positively to $\mathbb{E}(\Pi)$ is where $\mathbb{E}(b_{0ik} - b_{0jk}) < 0$ and $\mathbb{E}(b_{1ik} - b_{1jk}) > 0$. Their three corresponding pairs of possible scenarios $\mathbb{E}(\mathbf{b_0}')$ and its mirror image $\mathbb{E}(\mathbf{b_1}')$ are presented below. Note that the cases covered by $\mathbb{E}(\mathbf{b_0}')$ and $\mathbb{E}(\mathbf{b_1}')$ are the same as $\mathbb{E}(\mathbf{b_1})$ and $\mathbb{E}(\mathbf{b_0})$ respectively. As such, the nine scenarios represented by $\mathbb{E}(\mathbf{b_0'}) \rightarrow \mathbb{E}(\mathbf{b_1'})$ can also be similarly described.

$$\mathbb{E}(\mathbf{b_0}') = \mathbb{E}\begin{pmatrix} (b_{0ik} & - & b_{0jk}) & < 0 \\ + & & ++ & \\ -- & & - & \\ - & & + & \end{pmatrix} \rightarrow \mathbb{E}\begin{pmatrix} (b_{1ik} & - & b_{1jk}) & > 0 \\ ++ & & + & \\ - & & -- & \\ + & & - & \end{pmatrix} = \mathbb{E}(\mathbf{b_1'})$$

Taken together, the two preceding expressions $\mathbb{E}(\mathbf{b_0}) \rightarrow \mathbb{E}(\mathbf{b_1})$ and $\mathbb{E}(\mathbf{b_0'}) \rightarrow \mathbb{E}(\mathbf{b_1'})$ cover all possible scenarios pertaining to stock price reaction by a given pair n to common factors that enhances $\mathbb{E}(\Pi)$. These 18 scenarios can be categorized into *i) delayed reaction, ii) overreaction, and iii) mixed reaction.* First, delayed reaction includes four scenarios, shown below, where the direction of stock i and j factor sensitivities are similar both to each other and between time t and $t-1$. In this category, $\mathbb{E}(\Pi)$ is driven by discrepancies in the timeliness of component stock price reaction to common factors. This is reflected by stock $i(j)$ responding mainly to f_{kt} and stock $j(i)$ responding mainly to f_{kt-1}.

$$\begin{pmatrix} & \textbf{Delayed reaction} & \\ (++,+) & \rightarrow & (+,++) \\ (+,++) & \rightarrow & (++,+) \\ (--,-) & \rightarrow & (-,--) \\ (-,--) & \rightarrow & (--,-) \end{pmatrix}$$

Second, overreaction contains eight possible scenarios where $\mathbb{E}(\Pi)$ is driven by either or both[12] stocks i and j overreacting to f_{kt} and subsequently revising to f_{kt-1}. E.g $(++,+) \rightarrow (-,+)$ represents stock i overreacting to f_{kt} and partially readjusting to f_{kt-1}. Nothing specific can be said about stock j's price response.

$$\begin{pmatrix} & \textbf{Overreaction} & \\ (++,+) & \rightarrow & \begin{pmatrix} --,- \\ -,+ \end{pmatrix} \\ (+,++) & \rightarrow & \begin{pmatrix} -,-- \\ +,- \end{pmatrix} \\ (--,-) & \rightarrow & \begin{pmatrix} ++,+ \\ +,- \end{pmatrix} \\ (-,--) & \rightarrow & \begin{pmatrix} +,++ \\ -,+ \end{pmatrix} \end{pmatrix}$$

Third, mixed reaction contains six scenarios. But unlike its two counterparts, the component stocks (i, j) of pair n exhibit opposite sensitivities to f_{kt}. Consider $b_{0ik} > 0$ and $b_{0jk} < 0$,

[12] Hence this category contains twice as many scenarios as the delayed reaction category

which is denoted $(+,-)$. Its three corresponding scenarios are subjected to different interpretations: i) $(+,-) \rightarrow (++,+)$ suggests delayed reaction by stock i and overreaction by stock j, ii) $(+,-) \rightarrow (-,--)$ suggests overreaction by stock i and delayed reaction by stock j, and iii) $(+,-) \rightarrow (-,+)$ suggests overreaction by both component stocks. A similar description applies to $b_{0ik} < 0$ and $b_{0jk} > 0$.

$$\begin{pmatrix} & \textbf{Mixed reaction} & \\ (+,-) & \rightarrow & \begin{pmatrix} ++,+ \\ -,-- \\ -,+ \end{pmatrix} \\ (-,+) & \rightarrow & \begin{pmatrix} ++,+ \\ -,-- \\ +,- \end{pmatrix} \end{pmatrix}$$

To note, both $(+,-) \rightarrow (-,+)$ and $(-,+) \rightarrow (+,-)$ are not categorized as overreaction. Technically, both scenarios do generate pairs-trading profit. However, it is conceptually awkward to contemplate two stocks that historically display co-movement and yet exhibit opposite factor sensitivities. This relates to the issue of restricted versus unrestricted matching. If pairs are unrestrictedly matched, then it is probable for two stocks (i, j) to react differently to a given f_{kt} and still move closely together over time. Conversely, if the stock population is partitioned into (say) industry, beta, BMS, HML and/or leverage etc and matching occurs within stratified samples, it is less likely for two stocks (i, j) to exhibit opposing factor sensitivities to a given f_{kt}. This is especially if the factor is used to stratify the population in the first place.

The endeavor of this paper is to provide insights into the various potential sources of pairs-trading profit so as to acquire a better understanding of the nature of this popular Wall Street trading strategy. Whether or not certain scenarios covered by our model are practically relevant to pairs-traders is a separate question that in part depends on the actual setting of their pairs-trading parameters.

3 Empirical applications

In this section, we discuss two potential applications of our model. First, empirically measure and contrast the economic significance to $\mathbb{E}(\Pi)$ of component stock price reactions to common factors and idiosyncratic news. Second, incorporate the number and type of matching restrictions into the analysis of $\mathbb{E}(\Pi)$. This allows us to test of the sensitivity of various profit components to the number of matching restrictions. While we provide testable hypotheses in the relevant sections, length constraint stipulates that we have to investigate them in a separate paper, with the current paper focusing more on the background of pairs-trading, the derivation of our model and two potential applications in actual pairs-trading.

3.1 Estimation of pairs-trading profit components

The estimation is discussed assuming a market model specification for the return generating process. Consider the CRSP value-weighted index return r_{mt} as a proxy for the market factor return. We estimate the factor sensitivities from the following time series regressions:

$$\begin{aligned} r_{it} &= a_i + b_{0im} r_{mt} + b_{1im} r_{mt-1} + \varepsilon_{it} \\ r_{jt} &= a_j + b_{0jm} r_{mt} + b_{1jm} r_{mt-1} + \varepsilon_{jt} \end{aligned} \tag{9}$$

Trzcinka (1986) and Brown (1989) show that most of the co-movement in stock returns can be captured by a single factor. If r_{mt} suffices in explaining co-movements in (i, j), such that $\sigma_{\varepsilon_{it},\varepsilon_{jt}} = 0$, then r_{it} and r_{jt} can be estimated as single regressions. But if the residuals are contemporaneously correlated, then equation (9) should be estimated as a system using the seemingly-unrelated regression (SUR) procedure. The term σ_m^2 can be calculated directly from r_{mt}, and the average factor sensitivities are estimated from equation (9). This provides a measure of the contribution to $\mathbb{E}(\Pi)$ by the dynamics of price reaction to the market factor $\mathbb{E}[(b_{0im} - b_{0jm})(b_{1im} - b_{1jm})]\sigma_m^2$. The residuals $\varepsilon_{it}, \varepsilon_{jt}$ of equation (9) allow us to estimate both $\sigma_{\varepsilon_{it},\varepsilon_{it-1}}$ and $\sigma_{\varepsilon_{jt},\varepsilon_{jt-1}}$. Lastly, the pairwise dispersion in expected returns $(\overline{r}_i - \overline{r}_j)^2$ is estimated by a_i, a_j.

Our model stipulates that the expected profit for a N-pair portfolio originates from one of three sources, such that $\mathbb{E}(\Pi)$ in equation (8) is the sum of its three profit components. Thus, we can express the various profit components as a percentage of $\mathbb{E}(\Pi)$, which allows the economic significance of each profit source to be gauged. Chan and Hameed (2006), who study stock price synchronicity and the extent of analyst converge in emerging markets, specifically adjust for size-induced lead-lag effects. The latter is empirically well documented, with Chan (1993), Mech (1993), Badrinath et al (1995) providing different explanations for the return of large firms leading those of small firms. Accordingly, we can examine the sensitivity of profit components when a size-related matching restriction is imposed on pairs trading. Specifically, consider the following hypothesis:

Hypothesis 1: *Overreaction to idiosyncratic news and the gap in expected returns of component stocks pose a larger (smaller) contribution to* $\mathbb{E}(\Pi)$ *than lead-lag reaction to common factors when pairs-trading is performed with restricted matching based on size-sorted samples (unrestricted matching).*

3.2 Restricted versus unrestricted matching

In this section, we incorporates the number of matching restrictions into our model to analyze its potential impact on $\mathbb{E}(\Pi)$. If matching restrictions are imposed, then it is reason to assert that stocks which are grouped in the same stratified sample should possess similar characteristics, with similarity increasing in the number of restrictions. If firms are sorted based on industry and within industry further sorted into size, then it is likely for a given pair that is matched within an industry-size sorted sample to be (say) competitors[13], or supplier/customer relations[14] The degree of similarity also depends on the type of restrictions imposed e.g industry-sorted firms would be more similar that size-sorted firms.

The number of matching restrictions is an important parameter to analyze because it can resolve conflicting empirical evidence on the economic significance of various profit components. Consider two pairs-trader, where Trader 1 attributes most of the profitability to lead-lag dynamics to common factors, while Trader 2 finds that most of pairs-trading profit is driven by overreaction to idiosyncratic news. If Trader 1 does not impose any matching restrictions but Trader 2 does, the findings by both traders are not contradictory.[15]

Consider two competing firm i and j. We argue that good (bad) news specific to firm i may exert a negative (positive) flow-on effect on the price of firm j and vice versa, such that $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}} < 0$ and $\sigma_{\varepsilon_{jt},\varepsilon_{it-1}} < 0$. Conversely, for two collaborating firms e.g customer, supplier or simply one firm being a major shareholder of another firm, we assume good (bad) news specific to one firm to have a similar flow-on effect onto the other firm. This argument could apply in both directions between (say) Firm i and its supplier Firm j. It may only apply in one direction if Firm i is a major shareholder of Firm j, but not vice versa. For a pair of collaborative firms, we expect $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}} \geq 0$ and $\sigma_{\varepsilon_{jt},\varepsilon_{it-1}} \geq 0$,

Accordingly, if restricted matching is imposed during pairs-trading, then the assumption $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}} = \sigma_{\varepsilon_{jt},\varepsilon_{it-1}} = 0$ in the current model may need to be relaxed. This introduces $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}}$ and $\sigma_{\varepsilon_{jt},\varepsilon_{it-1}}$ into $\mathbb{E}(\Pi)$, as shown in equation (10). Their impact on pairs-trading profitability depends on the nature of the relationship between pairwise firms.

$$\begin{aligned}\mathbb{E}(\Pi) = -\sum_{n=ij}^{N}[\sum_{k=1}^{K}(\mathbb{E}(b_{0ik} - b_{0jk})(b_{1ik} - b_{1jk})\sigma^2_{f_k}) + (\overline{r}_i - \overline{r}_j)^2 \\ + (\sigma_{\varepsilon_{it},\varepsilon_{it-1}} + \sigma_{\varepsilon_{jt},\varepsilon_{jt-1}} - \sigma_{\varepsilon_{it},\varepsilon_{jt-1}} - \sigma_{\varepsilon_{jt},\varepsilon_{it-1}})] \qquad (10)\end{aligned}$$

Equation (10) shows that pairing up competing firms may have a negative effect on $\mathbb{E}(\Pi)$. This is because when p_{it-1} moves in one direction, the price gap widens. This is followed by p_{jt} moving in the opposite direction, widening the even further, thus depressing $\mathbb{E}(\Pi)$. It also shows that pairing up collaborative firms may enhance $\mathbb{E}(\Pi)$. Intuitively, if stock i moved in one direction at time $t-1$ and stock j reacts similarly but with a lag, this is akin to delayed reaction to a common factor. But since the "commonality" is manifested in a collaborative relation, this profit component is specific to the component stocks of given pairs, and does not apply across stocks in general. Lastly, the collaborative relationship could be one-sided e.g. Firm i holding shares in Firm j but not vice versa, such that either $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}}$ or $\sigma_{\varepsilon_{jt},\varepsilon_{it-1}}$ is relevant to $\mathbb{E}(\Pi)$.

[13]E.g Boeing v. Airbus; Citigroup v. HSBC; Google v. Yahoo; Bridgestone-Firestone v. Yokohama Wheels; Dell v. IBM etc

[14]E.g Boeing & Singapore Airline; Motorola & Telstra.

[15]While not directly related, this example is inspired by the conflicting results between Lo and MacKinlay (1990) and Jegadeesh and Titman (1995) on the source of contrarian profitability. The former attribute the majority of contrarian profits to lead-lag effects while the latter find that most of contrarian profits is driven by overreaction to idiosyncratic news.

For a given pair n, the component firms could either be unrelated, competitors or share a collaborative relationship. But in a sufficiently large N-pair portfolio, there is no reason for one relationship to dominate the other ex-ante across pairs. Denote the cross-sectional averages of the cross-serial covariance in idiosyncratic returns of component stocks of pair n for a N-pair portfolio as $\overline{\sigma}_{\varepsilon_{it},\varepsilon_{jt-1}}$ and $\overline{\sigma}_{\varepsilon_{it},\varepsilon_{jt-1}}$. Consider the large sample result specified in equation (11), which implies $\overline{\sigma}_{\varepsilon_{it},\varepsilon_{jt-1}} = \overline{\sigma}_{\varepsilon_{it},\varepsilon_{jt-1}} = 0$.

$$\lim_{N\to\infty} \sum_{n=ij}^{N} (\sigma_{\varepsilon_{it},\varepsilon_{jt-1}}) = \lim_{N\to\infty} \sum_{n=ij}^{N} (\sigma_{\varepsilon_{jt},\varepsilon_{it-1}}) = 0 \tag{11}$$

Unrestricted matching facilitates the consideration of a large sample N-pair portfolio during pairs trading. Thus in the limit, both $\sum_{n=ij}^{N}(\sigma_{\varepsilon_{it},\varepsilon_{jt-1}})$ and $\sum_{n=ij}^{N}(\sigma_{\varepsilon_{jt},\varepsilon_{it-1}})$ are expected to be trivial to $\mathbb{E}(\Pi)$. But if restricted matching is imposed, then the small sample arguments put forth at the beginning of this section should be considered in the analysis of $\mathbb{E}(\Pi)$. In sum, the issues raised in this sub-section leads to the following related hypotheses. These can be tested as a detailed empirical paper.

Hypothesis 2A (Small sample): *As the number of restrictions increases, cross-serial covariances in idiosyncratic returns will become economically significant in smaller stratified samples. But the impact on the* $\mathbb{E}(\Pi)$ *of a given stratified sample is unknown and depends on the nature of the relationship i.e. competitive or collaborative.*

Hypothesis 2B: (Large sample): *As the number of restrictions decreases, cross-serial covariances in idiosyncratic returns becomes economically trivial as* $N \to \infty$.

4 Concluding remarks

Although pairs-trading has been around on Wall Street for at least 20 years and is a popular and widely used trading strategy among investment banks and managed funds, the set of parameters that describes it and the source of its profitability remain elusive due to the lack of academic research. Gatev et al (2006) is the only study that examines the performance of pairs-trading while raising several main issues including matching restrictions, degree of similarity between component stocks, return autocorrelation, and risk factor exposures.

Our overall objective in this paper is to provide a structured analytical framework that encompasses the various important issues that are empirically addressed by Gatev et al (2006). Specifically, we derive a pairs-trading expected profit function in order to identify its various profit components. In our model, pairs-trading profitability stems from one of four potential sources: negative serial covariance in idiosyncratic returns; positive cross-serial covariance in idiosyncratic returns of collaborative firms; discrepancy in the unconditional expected return of component stocks and lead-lag effects in component stock price reaction to unexpected common factor realizations. For the latter source, our model encapsulates all

18 possible scenarios pertaining to dissimilar common factor price reaction dynamics between pairwise stocks.

We motivate our model with two potential applications. First, empirically measure and contrast the relative economic significance of various profit components. Second, and more importantly, establish a link between expected profit and the number of matching restrictions imposed during formation. This then allows us to analyze the sensitivity of various profit components to the number and type of matching restrictions. We hypothesize that in small samples, cross-serial covariance in idiosyncratic returns are economically significant, although their impact on $\mathbb{E}(\Pi)$ depends on the nature of the relationship between pairwise firms. We also hypothesize that these residual cross-serial covariance terms will become economically trivial in large samples as the number of restrictions decrease i.e equation (11). Both hypotheses are being investigated in separate concurrent empirical papers.

With our current and subsequent papers, we hope to take a first step towards understanding the interaction between the various parameters that describe a pairs-trading strategy and the various sources of its profitability. This allows us to move away from the ad-hoc setting of pairs-trading parameters that is currently being practised.[16] By offering a better understanding of the risks in and rewards for pairs-trading, we can begin to turn this popular trading strategy from an art to a science, as prior studies did with technical (price-volume) analysis, contrarian trading and momentum trading.

THE END

[16] This is why pairs-trading is often branded as statistical arbitrage.

References

[1] Avramov, D., Chordia, T., Goyal, A., 2006. Liquidity and autocorrelations in individual stock returns. Journal of Finance LXI, 2365-2394.

[2] Brown, D., Jennings, R., 1989. On technical analysis. Review of Financial Studies 2, 527-552.

[3] Badrinath, S., Kale, J., Noe, T., 1995. Of shepherds, sheeps, and cross-autocorrelation in equity returns. Review of Financial Studies 8, 401-430

[4] Brown, S., 1989. The number of factors in securities returns. Journal of Finance 44, 1247-1262.

[5] Chan, K., Hameed, A., 2006. Stock price synchronicity and analyst coverage in emerging markets. Journal of Financial Economics 80, 115-147.

[6] Chan, K., 1993. Imperfect infomration and cross-autocorrelation among strock returns. Journal of Finance 48, 1211-1230.

[7] Elliott, R., Van Der Hoek, J., Malcolm, W., 2005. Pairs-trading. Quantitative Finance 5, 271-276.

[8] Engle, R., Granger, C., 1987. Co-integration and error-correction representation, estimation and testing. Econometrica 55, 251-276.

[9] Easley, D., O'Hara, M., 1987. Price, trade size, and information in securities markets. Journal of Financial Economics 19, 69-90.

[10] Fama, E., French, K., 1996. Multifactor explanations of asset pricing anomalies. Journal of Finance 51, 131-155.

[11] Gatev, E., Goetzmann, W.N., Rouwenhorst, G.K., 2006. Pairs trading: Performance of a relative-value arbitrage rule. Review of Financial Studies 19, 797-827.

[12] Glosten, L., Milgrom, P., 1985. Bid, ask, and transaction prices in a specialist market with heterogeneously informed investors. Journal of Financial Economics 13, 71-100.

[13] Grundy, B., Martin, S., 2001. Understanding the nature of the risks and the source of the rewards to momentum investing. Review of Financial Studies 14, 29-78.

[14] Hasbrouck, J., 1991. Measuring the information content of stock trades. Journal of Finance 46, 179-207.

[15] Hasbrouck, J., 1993. Assessing the quality of a security market: A new approach to transaction cost measurement. Review of Financial Studies 6, 191-212.

[16] Hasbrouck, J., 1995. One security, many markets: Determining the contributions to price discovery. Journal of Finance 50, 1175-1199.

[17] Jegadeesh, N., 1990. Evidence of predictable behavior of securities returns. Journal of Finance 45, 881-898.

[18] Jegadeesh, N., Titman, S., 1993. Returns to buying winners and selling losers: Implications for stock market efficiency. Journal of Finance XLVIII, 65-91.

[19] Jegadeesh, N., Titman, S., 1995. Overreaction, delayed reaction and contrarian profits. Review of Financial Studies 8, 973-993.

[20] Lehmann, B., 1990. Fads, martingales and market efficiency. Quarterly Journal of Economics 105, 1-28.

[21] Lo, A., MacKinlay, C., 1990. When are contrarian profits due to market overreaction? Review of Financial Studies 3, 175-205.

[22] Mech, T.S., 1993. Portfolio return autocorrelation. Journal of Financial Economics 34, 307-344.

[23] Trzcinka, C, 1986. On the number of factors in the Arbitrage Pricing Model. Journal of Finance 41, 347-368.

[24] Wang, J., 1994. A model of competitive stock trading volume. Journal of Political Economy 102, 127168.