Understanding the risks in and rewards for pairs-trading

Michael T. Chng*

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Abstract

Contrarian trading has received ample academic attention over the past two decades. Ironically, pairs-trading, which is based on simple contrarian principles and has over 20-years history on Wall Street, has been largely ignored by finance academics. We derive an analytical pairs-trading expected profit function, which reveals four potential profit sources: negative serial covariance in idiosyncratic returns; positive cross-serial covariance in idiosyncratic returns of collaborative firms e.g. customer/supplier; a gap in the unconditional expected return of component stocks and lead-lag reaction to common factors. For the latter, our model encapsulates all 18 possible scenarios pertaining to dissimilar price reaction dynamics to common factors between pairwise stocks. We motivate our model with two potential applications. First, empirically measure and contrast economic significance of various profit components. Second, analyze the sensitivity of various profit components to the number and type of matching restrictions. By doing so, we offer a better understanding of the risks in and rewards for pairs-trading.

JEL classification: G14, G15.

Key words and phrases: common factors, negative serial covariance, trading strategy.

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*Corresponding author's contacts: Department of Finance, University of Melbourne, VIC 3010, Australia. Tel: 613-83447660. Email: mchng@unimelb.edu.au. Funding from the Melbourne Center for Financial Studies is gratefully acknowledged. I retain full property rights to all existing errors.

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1 Introduction

Pairs-trading is a form of technical analysis that is widely used among hedge funds and investment banks. The process involves exhaustively matching, identifying and ranking pairwise stocks (i, j) whose historical prices move closely together over time. After matched stocks are identified, the price gap $s_{nt} = p_{it} - p_{jt}$ between component stocks i and j of a given pair $n, n = 1, 2, ..., N$ acts as a signal to open and close a near market-neutral position. If s_{nt} exceeds a pre-specified threshold ε, a long-short position is created by short-selling the dearer stock to fund the purchase of the cheaper stock. Patterns in s_{nt} over time e.g negative serial covariance, implies positive expected profit from holding a zero-cost N-pair portfolio.

There are several reasons for the popularity of pairs-trading. First, the procedure is simple to understand and execute. Second, valuation models, which are subjected to wide error margins, are not required since pairs-trading is based on relative valuation and the position is often near market-neutral. Third, it is sufficiently flexible to accommodate various investment styles. Lastly, it normally does not evoke frequent intraday re-balancing, such that actual trading can be automated and is feasibly profitable.

Despite its history and popularity, the nature of pairs-trading remains elusive. It is commonly branded as statistical arbitrage, and yet pairs-trading relates to 'near' law-of-one-price, relative valuation, contrarian principles and cointegration. While its basic idea and procedures are straightforward, pairs-trading can be complicated by the calibration of a set of parameters. These include i) the matching criterion to identify pairwise stocks ii) whether to apply the matching criterion to the population of stocks i.e unrestricted matching, or to stratified samples i.e restricted matching iii) the type and number of restrictions iv) the threshold values for opening and closing positions v) pairs-portfolio size. In practice, these parameters are often determined in an ad-hoc fashion.

Pairs-trading is elusive due to the lack of academic research. Although it is based on simple contrarian principles, pairs-trading did not draw nearly as much academic attention as contrarian trading. To the best of our knowledge, Elliott et al (2005) and Gatev et at (2006) are the only two recent finance articles on pairs-trading. While the development of a structured framework that encompasses the various parameters of pairs-trading would no doubt attract practitioners' attention, that task is currently too complex. But a first step in that direction will require some understanding on the nature of pairs-trading. What are the risks involved? What are the sources of its rewards? How are the profit sources affected by the choice of parameters e.g. the types and/or number of restrictions. Price formation models, a cornerstone of the market microstructure literature, are the result of academic endeavors (Glosten and Milgrom (1985); Easley and O'Hara (1987); Brown and Jennings (1989); and Hasbrouck (1991, 1993, 1995)) to turn technical analysis from an art to a science.

Along similar lines, we take that first step to shed some academic light on the nature of pairs-trading. Gatev et al (2006) provide a detailed empirical analysis of the performance of a generic pairs-trading strategy. They perform a series of sensitivity analysis to address some important aspects of this trading strategy. Our overall objective in this paper is to derive a structured analytical framework that identifies the various components of expected pairs-trading profit $\mathbb{E}(\Pi)$. This allows us to address some of the main issues regarding pairs-trading, including those empirically examined by Gatev et al (2006), which we shall discuss shortly. More importantly, we establish a relationship between the sources of pairs-trading profit and the number of restrictions imposed during matching. Achieving this, we offer more insight into the risks in and rewards for pairs-trading. Our model reveals that expected pairs-trading profitability stems from one of four potential sources.

First, any stock price overreaction to firm-specific information induces negative serial covariance in idiosyncratic returns $\sigma_{\varepsilon_{it},\varepsilon_{it-1}} < 0$, which we show to increase $\mathbb{E}(\Pi)$. Gatev et al (2006) investigate this by contrasting between pairs-trading and contrarian trading to examine the role that return autocorrelations play in driving pairs-trading profit.

Second, we show that a discrepancy in the expected returns of component stocks $(\overline{r}_i - \overline{r}_j)^2$ is another component of $\mathbb{E}(\Pi)$. Intuitively, since opposite positions are to be taken in component stocks, the larger the gap between $\overline{r}_i$ and $\overline{r}_j$, the greater the difference in the risk factor exposures of stocks i and j, such that a long-short position created from pairwise stocks (i, j) is not market neutral. This net risk factor exposure(s) requires a higher $\mathbb{E}(\Pi)$. Gatev et al (2006) examine this by separately analyzing and contrasting results between the top ranked pairs 1-20 against the bottom ranked pairs 101-120. This they state is "valuable because most of the top pairs share certain characteristics...".

Third, Gatev et al (2006) consider five common factors to examine the risk characteristics of pairs-trading. In addition to the Fama and French (1996) β_m, HML and SMB factors, they also include a reversal factor and a momentum factor. Our model is derived with a multi-factor specification, which allows us to accommodate the Fama-French factors. Lead-lag effects in the common factor price reaction is incorporated into our model by allowing the realized return process to be exposed to both the contemporaneous and lagged factor realizations. We show that the common factor price reaction term in our model encapsulates all 18 scenarios pertaining tothe dynamics in component stock responses to common factors. They can be categorized as either delayed reaction or overreaction, or a mixture of both. These two categories covered by our model directly address the momentum and reversal factors that Gatev et al (2006) empirically examined.

Fourth, Gatev et al (2006) examine pairs-trading performance based on matching within S&P board industry groups,[1] and find robust pairs-trading profit across sector groups. In Section 3.2, we examine the impact on $\mathbb{E}(\Pi)$ associated with increasing the number of matching restrictions. We argue that firms allocated into smaller and smaller stratified samples are likely to possess increasingly similar characteristics. If this statement is reasonable, it implies that the zero cross-serial covariance in the idiosyncratic returns assumption in Jegadeesh and Titman (1995) should be relaxed. We define similarity between two firms in the context of either a competitive[2], or collaborative[3] relationship e.g. customer/supplier; one firm being a major shareholder in the other firm. We show how pairing up two competitors (collaborators) actually reduces (increases) $\mathbb{E}(\Pi)$.

Since the various components of $\mathbb{E}(\Pi)$) can be empirically estimated, our model provides a measure of the economic significance of various pairs-trading profit sources. More importantly, it allows us to analyze the sensitivity of various profit components when we vary the type and number of matching restrictions.[4] By doing so, we offer a better understanding of the risks in and rewards for pairs-trading.

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[1] They are Utilities, Transportation, Financial and Industrial

[2] E.g Boeing v. Airbus; Citigroup v. HSBC; Google v. Yahoo; Bridgestone-Firestone v. Yokohama Wheels; Dell v. IBM etc

[3] E.g Boeing & Singapore Airline; Motorola & Telstra.

[4] E.g Avramov et al (2006) document a strong (volume-adjusted) relationship between short-run return reversals and illiquidity. We can easily test the sensitivity of various pairs-trading profit components to liquidity-sorted stratified samples.

The description of the pairs-trading model is outlined in section 2. Empirical applications and hypotheses are discussed in section 3. Section 4 concludes.

2 The Sources of Pairs-trading Expected Profit

2.1 Background

Contrarian trading stipulates selling past winners and buying past loser stocks. Its execution normally involves ranking stocks based on their time t-1 returns, then take simultaneous long and short-sell positions in (say) the top decile (loser) and bottom decile (winner) portfolios and hold until time t. The strategy is designed to profit from overreaction and subsequent mean-reversion i.e. negative serial correlation in stock returns. Positive profits are reported in both Jegadeesh (1990) and Lehmann (1990).

However, Lo and MacKinlay (1990) show that contrarian profits could also be driven by delayed reaction or lead-lag effects between winner and loser stocks. In brief, if stock j reacts in the same direction as stock i but with a delay, then buying (selling) j subsequent to an increase (decrease) in i should generate profits, even if neither stocks overreact. Their results show that around 50% of contrarian profits is generated by such lead-lag effects. The essence of Lo and MacKinlay (1990) is to highlight both negative serial covariance $\sigma_{r^i_t, r^i_{t-1}} < 0$ and positive cross-serial covariance $\sigma_{r^i_t, r^j_{t-1}} > 0 \ \forall i \neq j$ in stock returns as two potential sources of contrarian profits.

Jegadeesh and Titman (1995) extends Lo and MacKinlay (1990) by associating lead-lag effects with the dynamics of price reaction to common factors. Their analysis of contrarian profits include a more detailed set of stock price reaction scenarios covering under and overreaction to common factors and idiosyncratic news. Unlike Lo and MacKinlay (1990), Jegadeesh and Titman (1995) find most of the contrarian profit is driven by overreaction to idiosyncratic news. This is consistent with the fact that overreaction to idiosyncratic news always generates contrarian profits, but overreaction to common factors may actually decrease contrarian profits. The essence of Jegadeesh and Titman (1995) is to show that common factor price reaction is a more appropriate measure of lead-lag effects than cross serial covariance in total returns.

For our pairs-trading model, we apply the Lo and MacKinlay portfolio weighting scheme and the Jegadeesh and Titman (1995) factor model based profit decomposition to identify various potential sources of pairs-trading expected profit. In a later section, we relax the Jegadeesh and Titman (1995) assumption of zero cross-serial covariance in idiosyncratic returns. In their model, co-movement in stocks is entirely driven by common factors. We explain in Section 3 that this assumption may not be reasonable when analyzing pairs-trading profitability since the trading strategy could involve pairwise firms that share an idiosyncratic relationship e.g. competitor, customer, supplier etc.

2.2 The nature of pairs-trading

Pairs-trading differs from contrarian trading in that it requires the matching of similar securities during the formation horizon $(T-t)$ prior to ranking.[5] Conceptually, by identifying and creating pairs of similar stocks, what pairs-trading generates and profits from is a set of

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[5] Here, ”similar” is defined in the context of the selected matching criterion e.g minimum Euclidean distance.

a synthetic securities[6] with prices $\{s_{1t}, s_{2t}, ..., s_{Nt}\}$ and $\mathbb{E}(s_n) = 0 \; \forall n$. But unlike the price of a tradable security, s_{nt} can be < 0. Since s_{nt} is denoted as $p_{it} - p_{jt}$, hence if $s_{nt} > (<) + (-)\varepsilon$, short-sell (buy) pair n at s_{nt}, which is equivalent to short-selling (buying) stock i and buying (short-selling) stock j.[7] It is a reasonable conjecture that, given its 20-year history, pairs-trading may actually be inspired by Engle and Granger (1987)'s paper on cointegration and error-correction between non-stationary time-series. To note, pairs-trading only requires $\mathbb{E}(s_n) = c$. Without loss of generality, we consider the simple case of $c = 0$.

A pairs-trading strategy requires the setting of an array of parameters listed below. In practise, these parameters are often determined in an ad-hoc fashion.[8] During formation, the population of listed firms is screened to remove any stocks that do not trade for one or more days. This removes from consideration illiquid stocks that hinders the actual trading of matched pairs. It also removes the moot outcome of creating pair whose component prices happen to be close and remain static during $(T - t)$.

1. A pre-specified (set of) matching criterion to identify pairwise stocks
2. Whether the matching criterion is to be applied to the population of stocks i.e unrestricted matching, or to stratified samples of stocks i.e restricted matching
3. The type and number of restrictions

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[6] When all possible pairwise combinations are considered, the number of synthetic securities outstrips the corresponding actual number of securities. For every N securities in a market, there are $\{p_{1t}, ...p_{Nt}\}$ prices and $\frac{N!}{2(N-2)!}$ price gaps. E.g. A stock market that contains 100 stocks has 100 price series and 100!/2*98!=4,950 price gap series. This implies more opportunities from trading pairs of securities than the securities themselves in (say) a valuation context.

[7] This perception is inspired by the pricing of a fixed-for-floating interest rate swap, where the seller of the swap is simultaneously selling the floating rate bond and buying the fixed rate bond, such that the swap price is the price differential between its fixed and floating components.

[8] To reiterate, the objective of our paper is to identify the various sources of pairs-trading profits. We address questions relating to the optimal setting of pairs-trading parameters in future research.

4. The threshold values for opening and closing positions

5. Pairs-portfolio size

There exists several criteria for identifying and creating pairwise stocks that are deemed close to each other. Gatev et al (2006) consider a simple criterion in minimizing the Euclidian distance between the normalized prices (including reinvested dividends) of exhaustively matched firms $\mathrm{Min}\sqrt{\sum_t^T (p_{it} - p_{jt})^2}$. This minimum distance criterion corresponds to the notion of pairing up stocks whose prices historically move closely together.[9] At the end of $(T-t)$, the various $\sqrt{\sum_t^T (s_{nt}^2)}$ are ranked to identify appropriate matched pairs $n = 1, 2, ..., N$ of similar stocks (i, j) for subsequent consideration in the trading horizon $(U - T)$. The latter commences the next trading day onwards. It is possible for the same stock to appear in two or more matched pairs that qualify for consideration during $(U - T)$. Subsequent empirical analysis could examine how the repeat appearance of a given stock affects profitability.

Another required parameter is a pre-specified threshold value ε that triggers actual trading in the component stocks of a given pair n during $(U - T)$. Like other parameters, there exists no paradigm for setting threshold values, nor whether the opening and closing thresholds should differ. Generally, a long-short position is created in the component stocks of pair n once $|s_{nT_1}| > \varepsilon$, which will be subsequently closed when $s_{nT_2} < \varepsilon$, $_{T<T_1<T_2<...<U}$.[10] Gatev et al (2006) specify a threshold that conforms with the adopted matching criterion. They set $|s_{nT_1}| > \varepsilon = 2\sqrt{\sum_t^T s_{nt}^2}$ as the opening trigger. An existing position in a given pair will only be closed out when $s_{nT_2} = 0$. If prices do not converge by the end of $(U - T)$, gains/losses will be calculated based on closing prices on the last trading day. As such, during $(U - T)$,

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[9] As a separate empirical paper, we investigate the profitability of various matching criteria

[10] As such, it is possible for a given pair n to trade numerous times during $(U - T)$.

a given pair n can be described by one of the following situations:

1. No trade whatsoever
2. Position is opened, but is closed by default at time U
3. Position is opened & closed once i.e. one complete cycle, then 1)
4. One complete cycle, then 2)
5. Similar to 3) or 4), but for two or more complete cycles

2.3 Decomposition of pairs-trading profits

Denote $r_{it} = p_{it} - p_{it-1}$, $\Delta s_{nt} = s_{nt} - s_{nt-1} = r_{it} - r_{jt}$ and $\Delta \overline{s}_t = \frac{1}{N}\sum_{n=1}^{N} \Delta s_{nt}$. Consider the following K-factor return generating processes of stock i and j respectively.

$$\begin{aligned} r_{it} &= \overline{r}_i + \sum_{k=1}^{K}(b_{0ik}f_{kt} + b_{1ik}f_{kt-1}) + \varepsilon_{it} \\ r_{jt} &= \overline{r}_j + \sum_{k=1}^{K}(b_{0jk}f_{kt} + b_{1jk}f_{kt-1}) + \varepsilon_{jt} \end{aligned} \tag{1}$$

The terms $\overline{r}_i$ and $\overline{r}_j$ are the unconditional expected returns of stock i and j respectively; f_{kt} is the unexpected k^{th}-factor realization i.e $\sigma_{f_{kt}f_{kt-1}} = 0$. The coefficients b_{0ik} and b_{1ik} are the sensitivities of stock i to the contemporaneous and lagged factor realizations. Lastly, ε_{it} and ε_{jt} are the corresponding idiosyncratic stock returns of i and j.

$$\mathbb{E}[\Delta s_{nt} - \Delta \overline{s}_t | \Delta s_{nt-1} - \Delta \overline{s}_{t-1} > 0] < 0$$

$$\mathbb{E}[\Delta s_{nt} - \Delta \overline{s}_t | \Delta s_{nt-1} - \Delta \overline{s}_{t-1} < 0] > 0$$

$$\therefore -\mathbb{E}[(\Delta s_{nt} - \Delta \overline{s}_t)(\Delta s_{nt-1} - \Delta \overline{s}_{t-1})] > 0 \tag{2}$$

The existence of pairs-trading profitability suggests that pairs which experience above average widening of their price gaps at time $t-1$ are expected to narrow their gaps at time t. Put differently, the superior performance of pairs-trading implies the condition specified in equation (2). We consider a weighting scheme similar to Lehmann (1990) and Lo and MacKinlay (1990) for two reasons. In addition to be being analytically tractable, the weighting scheme allows the expected profit function $\mathbb{E}(\Pi)$ to encompass equation (2).

$$w_{nt} = -\frac{1}{N}(\Delta s_{nt-1} - \Delta \overline{s}_{t-1}) \tag{3}$$

For a N-pair portfolio, the weight w_{nt} assign to pair n at time t is defined in equation (3). It takes into account the difference between the change in the gap Δs_{nt-1} and its cross-sectional mean $\Delta \overline{s}_{t-1}$ at time $t-1$. If s_{nt} exhibits a positive (negative) widening, the weight assigned to pair n would be a large negative (positive) value to indicate a short-sell (purchase) i.e. short-sell (buy) stock i and buy (short-sell) stock j. While w_{nt} is not necessarily zero, equation (4) shows that the total investment in a N-pair portfolio is zero by construction.

$$\sum_{n=1}^{N} w_{nt} = -\frac{1}{N}[\sum_{n=1}^{N} \Delta s_{nt-1} - N\Delta \overline{s}_{t-1}] = 0 \tag{4}$$

Equation (5) outlines the N-pair portfolio profit Π_t at time t. Similar to a portfolio return computation, Π_t is the cross-sectional weighted average change in price gap. If $\Delta s_{nt-1} < 0$, then $w_{nt} > 0$, and if $\Delta s_{nt} > 0$, Π_t increases. In words, a negative change in the price gap for pair n at time $t-1$ triggers a purchase. This is subsequently closed out at a profit when pair n exhibits a positive change i.e. the price gap narrows at time t, and vice versa.

$$\begin{aligned}\Pi_t = \sum_{n=1}^{N} w_{nt}\Delta s_{nt} &= -\frac{1}{N}[\sum_{n=1}^{N}(\Delta s_{nt-1} - \Delta\overline{s}_{t-1})\Delta s_{nt}] \\ &= -\frac{1}{N}[\sum_{n=1}^{N}(\Delta s_{nt}\Delta s_{nt-1}) - N\Delta\overline{s}_{t-1}\sum_{n=1}^{N}\Delta s_{nt}] \\ &= \frac{-\sum_{n=1}^{N}(\Delta s_{nt}\Delta s_{nt-1}) + \sum_{n=1}^{N}\Delta s_{nt}\sum_{n=1}^{N}\Delta s_{nt-1}}{N}\end{aligned} \tag{5}$$

Equation (5) allows us to obtain a general expression of the expected pairs-trading profit function $\mathbb{E}(\Pi)$ in equation (6). It shows that the N-pair portfolio's profitability is mainly driven by the component stocks' first order serial-cross serial covariance matrices of the various pairs in the portfolio. $\mathbb{E}(\Pi)$ also depends on the squared difference between $\overline{r}_i$ and $\overline{r}_j$. Intuitively, the larger the difference in the unconditional expected returns, the greater the tendency for matched component prices to diverge, which translates to greater potential profit from trading such paired stocks.

$$
\begin{aligned}
\mathbb{E}(\Pi) &= -\frac{\sum_{n=1}^{N}}{N}[\mathbb{E}(\Delta s_{nt}\Delta s_{nt-1}) - \mathbb{E}(\Delta s_{nt})\mathbb{E}(\Delta s_{nt-1})] \\
&= -\frac{\sum_{n=1}^{N}}{N}[\sigma_{\Delta s_{nt},\Delta s_{nt-1}}] \\
&= -\frac{\sum_{n=ij}^{N}}{N}[\begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}\sigma_{r_{it},r_{it-1}} & \sigma_{r_{jt},r_{it-1}} \\ \sigma_{r_{it},r_{jt-1}} & \sigma_{r_{jt},r_{jt-1}}\end{pmatrix}\begin{pmatrix}1 \\ -1\end{pmatrix} - (\overline{r}_i - \overline{r}_j)^2]
\end{aligned}
\tag{6}
$$

To gain more insight into the various sources of pairs-trading profitability, a decomposition of the various serial and cross-serial covariances terms is required. First, consider the serial covariance term $\sigma_{r_{it},r_{it-1}} = \mathbb{E}(r_{it}r_{it-1}) - \overline{r}_i^2$. Substituting the return generating processes in equation (1) into $\sigma_{r_{it},r_{it-1}}$ yields the following:

$$
\begin{aligned}
\sigma_{r_{it},r_{it-1}} &= \mathbb{E}(r_{it}r_{it-1}) - \overline{r}_i^2 \\
&= \mathbb{E}(\overline{r}_i^2 + \overline{r}_i\sum_{k=1}^{K}(b_{0ik}f_{kt-1} + b_{1ik}f_{kt-2}) + \overline{r}_i\varepsilon_{it-1} \\
&+ \overline{r}_i\sum_{k=1}^{K}(b_{0ik}f_{kt} + b_{1ik}f_{kt-1}) + \sum_{k=1}^{K}(b_{0ik}f_{kt} + b_{1ik}f_{kt-1})\sum_{k=1}^{K}(b_{0ik}f_{kt-1} + b_{1ik}f_{kt-2}) \\
&+ \varepsilon_{it-1}\sum_{k=1}^{K}(b_{0ik}f_{kt} + b_{1ik}f_{kt-1}) + \overline{r}_i\varepsilon_{it} + \varepsilon_{it}\sum_{k=1}^{K}(b_{0ik}f_{kt-1} + b_{1ik}f_{kt-2}) + \varepsilon_{it}\varepsilon_{it-1}) - \overline{r}_i^2
\end{aligned}
$$

Assume factors are orthogonal, such that $\sigma_{f_{lt}f_{mt}} = 0_{\ \forall l \neq m}$. And since f_{kt} is an unexpected factor realization, $\sigma_{f_{lt},f_{mt-1}} = 0_{\ \forall l \neq m}$ and $\mathbb{E}(f_{kt}^2) = \sigma_{f_k}^2$. Also, assume $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}} = 0_{\ \forall i \neq j}$ i.e no cross-serial covariances in idiosyncratic returns. Lastly, by definition, $\mathbb{E}(f_k) = \mathbb{E}(\overline{r}_i f_k) = \mathbb{E}(\overline{r}_i\varepsilon_{it}) = \mathbb{E}(f_{kt}\varepsilon_{it}) = 0$. Under these assumptions, $\sigma_{r_{it},r_{it-1}}$ collapses to equation (7), which shows that serial covariance in stock i returns is driven by the dynamics of expected price responses $\mathbb{E}(b_{0ik}b_{1ik})$ to common factors σ_{fk}^2 as well as serial covariance in idiosyncratic returns. If the price of stock i is expected to fully respond to $\sigma_{fk}^2 \forall k$ at either time t or $t-1$, then either $\mathbb{E}(b_{1ik})$ or $\mathbb{E}(b_{0ik}) = 0$, such that $\mathbb{E}(b_{0ik}b_{1ik}) = 0$. In addition, if idiosyncratic returns are serially uncorrelated i.e. $\sigma_{\varepsilon_{it}\varepsilon_{it-1}} = 0$, then $\sigma_{r_{it},r_{it-1}} = 0$.

$$\sigma_{r_{it},r_{it-1}} = \mathbb{E}[\sum_{k=1}^{K}(b_{0ik}b_{1ik}f_{kt-1}^2)] + \varepsilon_{it}\varepsilon_{it-1}) = \sum_{k=1}^{K}\mathbb{E}(b_{0ik}b_{1ik})\sigma_{f_k}^2 + \sigma_{\varepsilon_{it}\varepsilon_{it-1}} \tag{7}$$

To note, while $\sigma_{r_{jt},r_{jt-1}}$ can be similarly obtained, the off-diagonal cross serial covariances $\sigma_{r_{it},r_{jt-1}}$ and $\sigma_{r_{jt},r_{it-1}}$ are slightly different. Since idiosyncratic returns are cross-serially uncorrelated, if $\sigma_{r_{it},r_{jt-1}} \neq 0$, it can only be driven by stock j leading stock i in response to σ_{fk}^2. This is measured by $\mathbb{E}(b_{0jk}b_{1ik})$. Conversely, if the stock i price reaction to σ_{fk}^2 leads that of stock j, then $\sigma_{r_{jt},r_{it-1}} \neq 0$, which is measured by $\mathbb{E}(b_{0ik}b_{1jk})$. If neither stock is expected to react with a delay, $\mathbb{E}(b_{1ik}) = \mathbb{E}(b_{1jk}) = 0$. If both stocks are expected to react with a delay, $\mathbb{E}(b_{0ik}) = \mathbb{E}(b_{0jk}) = 0$. In both cases, $\sigma_{r_{it},r_{jt-1}} = \sigma_{r_{jt},r_{it-1}} = 0$ i.e. no cross-serial correlation whatsoever. Lastly, if both stocks i and j exhibit dynamics in their price responses to σ_{fk}^2, then both $\sigma_{r_{jt},r_{it-1}}$ and $\sigma_{r_{it},r_{jt-1}}$ are non-zero i.e. there is bi-directional causality. The following summarizes.

$$\begin{pmatrix}\sigma_{r_{it},r_{it-1}} & \sigma_{r_{jt},r_{it-1}} \\ \sigma_{r_{it},r_{jt-1}} & \sigma_{r_{jt},r_{jt-1}}\end{pmatrix} = \sum_{k=1}^{K}\begin{pmatrix}b_{0ik}b_{1ik} & b_{0ik}b_{1jk} \\ b_{0jk}b_{1ik} & b_{0jk}b_{1jk}\end{pmatrix}\sigma_{f_k}^2 + \begin{pmatrix}\sigma_{\varepsilon_{it},\varepsilon it-1} & 0 \\ 0 & \sigma_{\varepsilon_{jt},\varepsilon_{jt-1}}\end{pmatrix}$$

$$= \begin{pmatrix}b_{0i1} & \dots & b_{0iK} \\ b_{0j1} & \dots & b_{0jK}\end{pmatrix} \times \begin{pmatrix}\sigma_{f_1}^2 & 0 & 0\dots & \\ 0 & \sigma_{f_2}^2 & 0\dots & \\ . & . & . & \\ 0 & 0 & 0\dots & \sigma_{f_K}^2\end{pmatrix} \times \begin{pmatrix}b_{1i1} & b_{1j1} \\ b_{1i2} & b_{1j2} \\ . & . \\ & \\ b_{1iK} & b_{1jK}\end{pmatrix} + \begin{pmatrix}\sigma_{\varepsilon_{it},\varepsilon it-1} & 0 \\ 0 & \sigma_{\varepsilon_{jt},\varepsilon_{jt-1}}\end{pmatrix}$$

Substituting the above into equation (6) leads to a more detailed expression for $\mathbb{E}(\Pi)$, which is outlined in equation (8). It shows that pairs-trading $\mathbb{E}(\Pi)$ comes from one of three potential sources. First, it is driven by negative serial correlation in the idiosyncratic returns of either or both stocks i and j since $\sigma_{\varepsilon_{it},\varepsilon_{it-1}} < 0$ and/or $\sigma_{\varepsilon_{jt},\varepsilon_{jt-1}} < 0$ increases $\mathbb{E}(\Pi)$. This shows that profitability of pairs-trading is potentially driven by overreaction to firm-specific news. Second $\mathbb{E}(\Pi)$ is driven by a gap between $\overline{r}_i$ and $\overline{r}_j$. To reiterate, pairs are formed based on their historical prices having moved closely together. A non-zero gap in the unconditional expected returns implies a tendency for prices to diverge. Taken together, this translates to negative serial correlation in s_{nt} over time, which pairs-trading profitability depends upon.

$$\begin{aligned}
\mathbb{E}(\Pi) = & -\sum_{n=ij}^{N}[\sum_{k=1}^{K}[\mathbb{E}(b_{0ik}b_{1ik}) + \mathbb{E}(b_{0jk}b_{1jk})]\sigma^2_{f_k} - \sum_{k=1}^{K}[\mathbb{E}(b_{0ik}b_{1jk}) + \mathbb{E}(b_{0jk}b_{1ik})]\sigma^2_{f_k}] \\
& + \sigma_{\varepsilon_{it},\varepsilon_{it-1}} + \sigma_{\varepsilon_{jt},\varepsilon_{jt-1}} - (\overline{r}_i - \overline{r}_j)^2) \\
= & -\sum_{n=ij}^{N}\sum_{k=1}^{K}(\mathbb{E}(b_{0ik} - b_{0jk})(b_{1ik} - b_{1jk}))\sigma^2_{f_k} - \sum_{n=ij}^{N}((\sigma_{\varepsilon_{it},\varepsilon_{it-1}} + \sigma_{\varepsilon_{jt},\varepsilon_{jt-1}}) - (\overline{r}_i - \overline{r}_j)^2)
\end{aligned} \tag{8}$$

Third, $\mathbb{E}(\Pi)$ depends on the price reaction dynamics of the two component stocks (i, j) to common factors. This is encompassed by the first term of equation (8), which has three components: $\mathbb{E}(b_{0ik} - b_{0jk}), \mathbb{E}(b_{1ik} - b_{1jk})$ and $\sigma^2_{f_k} > 0$. Accordingly, for the timeliness of stock price reaction to common factors to impact positively on $\mathbb{E}(\Pi)$, this requires $\mathbb{E}[(b_{0ik} - b_{0jk})(b_{1ik} - b_{1jk})] < 0$, which in turn requires either $\mathbb{E}(b_{0ik} - b_{0jk}) > 0$ and $\mathbb{E}(b_{1ik} - b_{1jk}) < 0$, or vice versa. $\mathbb{E}(\mathbf{b_0})$ and its mirror image $\mathbb{E}(\mathbf{b_1})$ each lists three pairs of possible scenarios for $\mathbb{E}(b_{0ik} - b_{0jk}) > 0$ and $\mathbb{E}(b_{1ik} - b_{1jk}) < 0$ respectively.

$$\mathbb{E}(\mathbf{b_0}) = \mathbb{E}\begin{pmatrix} (b_{0ik} & - & b_{0jk}) & > 0 \\ ++ & & + & \\ - & & -- & \\ + & & - & \end{pmatrix} \rightarrow \mathbb{E}\begin{pmatrix} (b_{1ik} & - & b_{1jk}) & < 0 \\ + & & ++ & \\ -- & & - & \\ - & & + & \end{pmatrix} = \mathbb{E}(\mathbf{b_1})$$

A few notes on how to read the above expression. First, $-b_{1ik}$ and $+b_{1jk}$ indicate that $b_{1ik} < 0, b_{1jk} > 0$, such that $(b_{1ik} - b_{1jk}) < 0$ regardless of the size of the coefficients. The case $++\,b_{0ik}$ and $+b_{0jk}$ indicate that while both coefficients are positive, $b_{0ik} > b_{0jk}$, such that $(b_{0ik} - b_{0jk}) > 0$. Second, $\mathbb{E}(\mathbf{b_0}) \rightarrow \mathbb{E}(\mathbf{b_1})$ represents nine possible scenarios that generate pairs-trading profit. An example of a delayed reaction scenario is $(++, +) \rightarrow (+, ++)$. Here most of the r_{it} common factor reaction occurs contemporaneously, while stock j reacts primarily to f_{kt-1}. The case $(-, --) \rightarrow (--, -)$ can be described in a similar fashion. The scenarios $(++, +) \rightarrow (--, -)$ and $(--, -) \rightarrow (++, +)$ represent overreaction by both r_{it} and r_{jt} to f_{kt}, with both component stocks subsequently revising to f_{kt-1}. Overreaction by pairwise stocks to a common factor does not imply profitability per se. The widening and subsequent narrowing of s_{nt} occurs if $r_{it}(r_{jt})$ overreacts by more than $r_{jt}(r_{it})$ to f_{kt}, but subsequently also revises by more than $r_{jt}(r_{it})$ to f_{kt-1}.

The converse situation that also contributes positively to $\mathbb{E}(\Pi)$ is where $\mathbb{E}(b_{0ik} - b_{0jk}) < 0$ and $\mathbb{E}(b_{1ik} - b_{1jk}) > 0$. Their three corresponding pairs of possible scenarios $\mathbb{E}(\mathbf{b_0}')$ and its mirror image $\mathbb{E}(\mathbf{b_1}')$ are presented below. Note that the cases covered by $\mathbb{E}(\mathbf{b_0}')$ and $\mathbb{E}(\mathbf{b_1}')$ are the same as $\mathbb{E}(\mathbf{b_1})$ and $\mathbb{E}(\mathbf{b_0})$ respectively. As such, the nine scenarios represented by $\mathbb{E}(\mathbf{b'_0}) \rightarrow \mathbb{E}(\mathbf{b'_1})$ can also be similarly described.

$$\mathbb{E}(\mathbf{b_0}') = \mathbb{E}\begin{pmatrix} (b_{0ik} & - & b_{0jk}) & < 0 \\ + & & ++ & \\ -- & & - & \\ - & & + & \end{pmatrix} \rightarrow \mathbb{E}\begin{pmatrix} (b_{1ik} & - & b_{1jk}) & > 0 \\ ++ & & + & \\ - & & -- & \\ + & & - & \end{pmatrix} = \mathbb{E}(\mathbf{b'_1})$$

Taken together, the two preceding expressions $\mathbb{E}(\mathbf{b_0}) \rightarrow \mathbb{E}(\mathbf{b_1})$ and $\mathbb{E}(\mathbf{b'_0}) \rightarrow \mathbb{E}(\mathbf{b'_1})$ cover all possible scenarios pertaining to stock price reaction by a given pair n to common factors that enhances $\mathbb{E}(\Pi)$. These 18 scenarios can be categorized into *i) delayed reaction, ii) overreaction, and iii) mixed reaction.* First, delayed reaction includes four scenarios, shown below, where the direction of stock i and j factor sensitivities are similar both to each other and between time t and $t-1$. In this category, $\mathbb{E}(\Pi)$ is driven by discrepancies in the timeliness of component stock price reaction to common factors. This is reflected by stock $i(j)$ responding mainly to f_{kt} and stock $j(i)$ responding mainly to f_{kt-1}.

$$\begin{pmatrix} & \textbf{Delayed reaction} & \\ (++,+) & \rightarrow & (+,++) \\ (+,++) & \rightarrow & (++,+) \\ (--,-) & \rightarrow & (-,--) \\ (-,--) & \rightarrow & (--,-) \end{pmatrix}$$

Second, overreaction contains eight possible scenarios where $\mathbb{E}(\Pi)$ is driven by either or both[11] stocks i and j overreacting to f_{kt} and subsequently revising to f_{kt-1}. E.g $(++,+) \rightarrow (-,+)$ represents stock i overreacting to f_{kt} and partially readjusting to f_{kt-1}. Nothing specific can be said about stock j's price response.

$$\begin{pmatrix} & \textbf{Overreaction} & \\ (++,+) & \rightarrow & \begin{pmatrix} --,- \\ -,+ \end{pmatrix} \\ (+,++) & \rightarrow & \begin{pmatrix} -,-- \\ +,- \end{pmatrix} \\ (--,-) & \rightarrow & \begin{pmatrix} ++,+ \\ +,- \end{pmatrix} \\ (-,--) & \rightarrow & \begin{pmatrix} +,++ \\ -,+ \end{pmatrix} \end{pmatrix}$$

Third, mixed reaction contains six scenarios. But unlike its two counterparts, the component stocks (i,j) of pair n exhibit opposite sensitivities to f_{kt}. Consider $b_{0ik} > 0$ and $b_{0jk} < 0$, which is denoted $(+,-)$. Its three corresponding scenarios are subjected to different interpretations: i) $(+,-) \rightarrow (++,+)$ suggests delayed reaction by stock i and overreaction by stock j, ii) $(+,-) \rightarrow (-,--)$ suggests overreaction by stock i and delayed reaction by stock j, and iii) $(+,-) \rightarrow (-,+)$ suggests overreaction by both component stocks. A similar description

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[11] Hence this category contains twice as many scenarios as the delayed reaction category

applies to $b_{0ik} < 0$ and $b_{0jk} > 0$.

$$\begin{pmatrix} & \textbf{Mixed reaction} & \\ (+,-) & \rightarrow & \begin{pmatrix} ++,+ \\ -,-- \\ -,+ \end{pmatrix} \\ & & \\ (-,+) & \rightarrow & \begin{pmatrix} ++,+ \\ -,-- \\ +,- \end{pmatrix} \end{pmatrix}$$

To note, both $(+,-) \rightarrow (-,+)$ and $(-,+) \rightarrow (+,-)$ are not categorized as overreaction. Technically, both scenarios do generate pairs-trading profit. However, it is conceptually awkward to contemplate two stocks that historically display co-movement and yet exhibit opposite factor sensitivities. This relates to the issue of restricted versus unrestricted matching. If pairs are unrestrictedly matched, then it is probable for two stocks (i, j) to react differently to a given f_{kt} and still move closely together over time. Conversely, if the stock population is partitioned into (say) industry, beta, BMS, HML and/or leverage etc and matching occurs within stratified samples, it is less likely for two stocks (i, j) to exhibit opposing factor sensitivities to a given f_{kt}. This is especially if the factor is used to stratify the population in the first place.

The endeavor of this paper is to provide insights into the various potential sources of pairs-trading profit so as to acquire a better understanding of the nature of this popular Wall Street trading strategy. Whether or not certain scenarios covered by our model are practically relevant to pairs-traders is a separate question that in part depends on the actual setting of their pairs-trading parameters.

3 Empirical applications

In this section, we discuss two potential applications of our model. First, empirically measure and contrast the economic significance to $\mathbb{E}(\Pi)$ of component stock price reactions to common factors and idiosyncratic news. Second, incorporate the number and type of matching restrictions into the analysis of $\mathbb{E}(\Pi)$. This allows us to test of the sensitivity of various profit components to the number of matching restrictions. While we provide testable hypotheses in the relevant sections, length constraint stipulates that we have to investigate them in a separate paper, with the current paper focusing more on the background of pairs-trading, the derivation of our model and two potential applications in actual pairs-trading.

3.1 Estimation of pairs-trading profit components

The estimation is discussed assuming a market model specification for the return generating process. Consider the CRSP value-weighted index return r_{mt} as a proxy for the market factor return. We estimate the factor sensitivities from the following time series regressions:

$$\begin{aligned} r_{it} &= a_i + b_{0im} r_{mt} + b_{1im} r_{mt-1} + \varepsilon_{it} \\ r_{jt} &= a_j + b_{0jm} r_{mt} + b_{1jm} r_{mt-1} + \varepsilon_{jt} \end{aligned} \tag{9}$$

Trzcinka (1986) and Brown (1989) show that most of the co-movement in stock returns can be captured by a single factor. If r_{mt} suffices in explaining co-movements in (i, j), such that $\sigma_{\varepsilon_{it},\varepsilon_{jt}} = 0$, then r_{it} and r_{jt} can be estimated as single regressions. But if the residuals are contemporaneously correlated, then equation (9) should be estimated as a system using the seemingly-unrelated regression (SUR) procedure. The term σ_m^2 can be calculated directly from r_{mt}, and the average factor sensitivities are estimated from equation (9). This provides a measure of the contribution to $\mathbb{E}(\Pi)$ by the dynamics of price reaction to the market factor $\mathbb{E}[(b_{0im} - b_{0jm})(b_{1im} - b_{1jm})]\sigma_m^2$. The residuals $\varepsilon_{it}, \varepsilon_{jt}$ of equation (9) allow us to estimate both $\sigma_{\varepsilon_{it},\varepsilon_{it-1}}$ and $\sigma_{\varepsilon_{jt},\varepsilon_{jt-1}}$. Lastly, the pairwise dispersion in expected returns $(\overline{r}_i - \overline{r}_j)^2$ is estimated by a_i, a_j.

Our model stipulates that the expected profit for a N-pair portfolio originates from one of three sources, such that $\mathbb{E}(\Pi)$ in equation (8) is the sum of its three profit components. Thus, we can express the various profit components as a percentage of $\mathbb{E}(\Pi)$, which allows the economic significance of each profit source to be gauged. Chan and Hameed (2006), who study stock price synchronicity and the extent of analyst converge in emerging markets, specifically adjust for size-induced lead-lag effects. The latter is empirically well documented, with Chan (1993), Mech (1993), Badrinath et al (1995) providing different explanations for the return of large firms leading those of small firms. Accordingly, we can examine the sensitivity of profit components when a size-related matching restriction is imposed on pairs trading. Specifically, consider the following hypothesis:

Hypothesis 1: *Overreaction to idiosyncratic news and the gap in expected returns of component stocks pose a larger (smaller) contribution to* $\mathbb{E}(\Pi)$ *than lead-lag reaction to common factors when pairs-trading is performed with restricted matching based on size-sorted samples (unrestricted matching).*

3.2 Restricted versus unrestricted matching

In this section, we incorporates the number of matching restrictions into our model to analyze its potential impact on $\mathbb{E}(\Pi)$. If matching restrictions are imposed, then it is reason to assert that stocks which are grouped in the same stratified sample should possess similar characteristics, with similarity increasing in the number of restrictions. If firms are sorted based on industry and within industry further sorted into size, then it is likely for a given pair that is matched within an industry-size sorted sample to be (say) competitors[12], or supplier/customer relations[13] The degree of similarity also depends on the type of restrictions imposed e.g industry-sorted firms would be more similar that size-sorted firms.

The number of matching restrictions is an important parameter to analyze because it can resolve conflicting empirical evidence on the economic significance of various profit components. Consider two pairs-trader, where Trader 1 attributes most of the profitability to lead-lag dynamics to common factors, while Trader 2 finds that most of pairs-trading profit is driven by overreaction to idiosyncratic news. If Trader 1 does not impose any matching restrictions but Trader 2 does, the findings by both traders are not contradictory.[14]

Consider two competing firm i and j. We argue that good (bad) news specific to firm i may exert a negative (positive) flow-on effect on the price of firm j and vice versa, such that $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}} < 0$ and $\sigma_{\varepsilon_{jt},\varepsilon_{it-1}} < 0$. Conversely, for two collaborating firms e.g customer, supplier or simply one firm being a major shareholder of another firm, we assume good (bad) news specific to one firm to have a similar flow-on effect onto the other firm. This argument could apply in both directions between (say) Firm i and its supplier Firm j. It may only apply in one direction if Firm i is a major shareholder of Firm j, but not vice versa. For a pair of collaborative firms, we expect $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}} \geq 0$ and $\sigma_{\varepsilon_{jt},\varepsilon_{it-1}} \geq 0$,

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[12] E.g Boeing v. Airbus; Citigroup v. HSBC; Google v. Yahoo; Bridgestone-Firestone v. Yokohama Wheels; Dell v. IBM etc

[13] E.g Boeing & Singapore Airline; Motorola & Telstra.

[14] While not directly related, this example is inspired by the conflicting results between Lo and MacKinlay (1990) and Jegadeesh and Titman (1995) on the source of contrarian profitability. The former attribute the majority of contrarian profits to lead-lag effects while the latter find that most of contrarian profits is driven by overreaction to idiosyncratic news.

Accordingly, if restricted matching is imposed during pairs-trading, then the assumption $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}} = \sigma_{\varepsilon_{jt},\varepsilon_{it-1}} = 0$ in the current model may need to be relaxed. This introduces $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}}$ and $\sigma_{\varepsilon_{jt},\varepsilon_{it-1}}$ into $\mathbb{E}(\Pi)$, as shown in equation (10). Their impact on pairs-trading profitability depends on the nature of the relationship between pairwise firms.

$$\begin{aligned}\mathbb{E}(\Pi) = -\sum_{n=ij}^{N}[\sum_{k=1}^{K}(\mathbb{E}(b_{0ik} - b_{0jk})(b_{1ik} - b_{1jk})\sigma^2_{f_k}) + (\overline{r}_i - \overline{r}_j)^2 \\ + (\sigma_{\varepsilon_{it},\varepsilon_{it-1}} + \sigma_{\varepsilon_{jt},\varepsilon_{jt-1}} - \sigma_{\varepsilon_{it},\varepsilon_{jt-1}} - \sigma_{\varepsilon_{jt},\varepsilon_{it-1}})] \qquad (10)\end{aligned}$$

Equation (10) shows that pairing up competing firms may have a negative effect on $\mathbb{E}(\Pi)$. This is because when p_{it-1} moves in one direction, the price gap widens. This is followed by p_{jt} moving in the opposite direction, widening the even further, thus depressing $\mathbb{E}(\Pi)$. It also shows that pairing up collaborative firms may enhance $\mathbb{E}(\Pi)$. Intuitively, if stock i moved in one direction at time $t-1$ and stock j reacts similarly but with a lag, this is akin to delayed reaction to a common factor. But since the "commonality" is manifested in a collaborative relation, this profit component is specific to the component stocks of given pairs, and does not apply across stocks in general. Lastly, the collaborative relationship could be one-sided e.g. Firm i holding shares in Firm j but not vice versa, such that either $\sigma_{\varepsilon_{it},\varepsilon_{jt-1}}$ or $\sigma_{\varepsilon_{jt},\varepsilon_{it-1}}$ is relevant to $\mathbb{E}(\Pi)$.

For a given pair n, the component firms could either be unrelated, competitors or share a collaborative relationship. But in a sufficiently large N-pair portfolio, there is no reason for one relationship to dominate the other ex-ante across pairs. Denote the cross-sectional averages of the cross-serial covariance in idiosyncratic returns of component stocks of pair n for a N-pair portfolio as $\overline{\sigma}_{\varepsilon_{it},\varepsilon_{jt-1}}$ and $\overline{\sigma}_{\varepsilon_{it},\varepsilon_{jt-1}}$. Consider the large sample result specified in equation (11), which implies $\overline{\sigma}_{\varepsilon_{it},\varepsilon_{jt-1}} = \overline{\sigma}_{\varepsilon_{it},\varepsilon_{jt-1}} = 0$.

$$\lim_{N\to\infty} \sum_{n=ij}^{N} (\sigma_{\varepsilon_{it},\varepsilon_{jt-1}}) = \lim_{N\to\infty} \sum_{n=ij}^{N} (\sigma_{\varepsilon_{jt},\varepsilon_{it-1}}) = 0 \tag{11}$$

Unrestricted matching facilitates the consideration of a large sample N-pair portfolio during pairs trading. Thus in the limit, both $\sum_{n=ij}^{N}(\sigma_{\varepsilon_{it},\varepsilon_{jt-1}})$ and $\sum_{n=ij}^{N}(\sigma_{\varepsilon_{jt},\varepsilon_{it-1}})$ are expected to be trivial to $\mathbb{E}(\Pi)$. But if restricted matching is imposed, then the small sample arguments put forth at the beginning of this section should be considered in the analysis of $\mathbb{E}(\Pi)$. In sum, the issues raised in this sub-section leads to the following related hypotheses. These can be tested as a detailed empirical paper.

Hypothesis 2A (Small sample): *As the number of restrictions increases, cross-serial covariances in idiosyncratic returns will become economically significant in smaller stratified samples. But the impact on the* $\mathbb{E}(\Pi)$ *of a given stratified sample is unknown and depends on the nature of the relationship i.e. competitive or collaborative.*

Hypothesis 2B: (Large sample): *As the number of restrictions decreases, cross-serial covariances in idiosyncratic returns becomes economically trivial as* $N \to \infty$.

4 Concluding remarks

Although pairs-trading has been around on Wall Street for at least 20 years and is a popular and widely used trading strategy among investment banks and managed funds, the set of parameters that describes it and the source of its profitability remain elusive due to the lack of academic research. Gatev et al (2006) is the only study that examines the performance of pairs-trading while raising several main issues including matching restrictions, degree of similarity between component stocks, return autocorrelation, and risk factor exposures.

Our overall objective in this paper is to provide a structured analytical framework that encompasses the various important issues that are empirically addressed by Gatev et al (2006). Specifically, we derive a pairs-trading expected profit function in order to identify its various profit components. In our model, pairs-trading profitability stems from one of four potential sources: negative serial covariance in idiosyncratic returns; positive cross-serial covariance in idiosyncratic returns of collaborative firms; discrepancy in the unconditional expected return of component stocks and lead-lag effects in component stock price reaction to unexpected common factor realizations. For the latter source, our model encapsulates all 18 possible scenarios pertaining to dissimilar common factor price reaction dynamics between pairwise stocks.

We motivate our model with two potential applications. First, empirically measure and contrast the relative economic significance of various profit components. Second, and more importantly, establish a link between expected profit and the number of matching restrictions imposed during formation. This then allows us to analyze the sensitivity of various profit components to the number and type of matching restrictions. We hypothesize that in small samples, cross-serial covariance in idiosyncratic returns are economically significant, although their impact on $\mathbb{E}(\Pi)$ depends on the nature of the relationship between pairwise firms. We also hypothesize that these residual cross-serial covariance terms will become economically trivial in large samples as the number of restrictions decrease i.e equation (11). Both hypotheses are being investigated in separate concurrent empirical papers.

With our current and subsequent papers, we hope to take a first step towards understanding the interaction between the various parameters that describe a pairs-trading strategy and the various sources of its profitability. This allows us to move away from the ad-hoc setting of pairs-trading parameters that is currently being practised.[15] By offering a better understanding of the risks in and rewards for pairs-trading, we can begin to turn this popular trading strategy from an art to a science, as prior studies did with technical (price-volume) analysis, contrarian trading and momentum trading.

THE END

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[15] This is why pairs-trading is often branded as statistical arbitrage.

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