Stress-Testing Credit Risk Parameters - An Application to Retail Loan Portfolios*

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Abstract

Financial institutions are faced with the challenge to forecast future credit portfolio losses. It is common practice to focus on portfolio models consisting of a limited set of parameters, such as the probability of default, asset correlation, loss given default or exposure at default. A simple portfolio model is also used in the Basel II framework for calculating regulatory capital. With regard to the stability of the financial system, these models have to be approved by regulators who have an interest in a conservative assessment of the credit portfolio risk and require the stress-testing of risk estimates.

The present paper is the first in its kind to develop a framework to stress the smallest building block, the sensitivities of risk drivers and therefore any derivative such as a risk parameter or the credit portfolio loss. As a result, estimation uncertainties as well as the correlations are taken into account. In an empirical analysis, the stress scenarios for different loan categories are analyzed for US retail borrowers and the implications on economic as well as regulatory capital explored.

Keywords: Basel II, Business Cycle, Capital Adequacy, Correlation, Credit Risk, Default Probability, Expected Loss, Stress-test, Value-at-Risk

JEL Classification: G20, G28, C51

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1 Introduction

This article is motivated by the need of financial institutions to forecast future credit losses for loan portfolios. It is common practice to quantify loan losses by a limited set of parameters, such as the probability of default (PD), asset correlation, loss given default (LGD) or exposure at default (EAD). In the new regulatory framework also known as Basel II, banks are allowed to determine some of these parameters (namely PD and LGD) using internal risk models. The specification of these parameters and the capital charges reflect the regulators' interest in a conservative assessment of the credit portfolio risk. Thus, the regulatory capital formula is based on stressed assumptions of different concepts, such as the

- Stress of PDs: PDs are stressed by a one-factor non-linear model where the factor equals the 99.9th percentile of a systematic standard normally distributed variable and the sensitivity is based on the so called asset correlation;
- Stress of EADs and LGDs: EADs and LGDs are modeled based on an economic downturn condition. This scenario is currently not specified and has recently caused confusion in the industry; and
- Stress of asset correlations: Asset correlations can be interpreted as a measure of the sensitivity of PDs to the business cycle and therefore have a major impact on the stress of the PDs. As a matter of fact, the Basel Committee specifies values which are significantly higher than empirical estimates observed in a couple of empirical studies (see e.g., [4], [17] and [18]). Examples are 15% for residential mortgage, 4% for qualifying revolving, between 16% and 3% (decreasing function of the probability of default) for other retail and between 24% and 12% (decreasing function of the probability of default) for corporate exposures.

Besides the calculation of regulatory capital the Basel concept is adopted by financial institutions for other applications such as the determination of economic capital, loan approval decisions, loan pricing, or the derivation of concentration limits. This approach can be problematic since it does not take estimation risk and interdependencies between the risk parameters and their drivers into account. As a result, it may lead to incorrect forecasts of the loss distribution and the derived capital allocation. Previous studies have shown that risk parameters such as default probabilities and losses given default are positively correlated (compare [1] or [18]). The same may be true for other risk parameters. In addition, credit risk models can be based on a variety of risk drivers and as a consequence may already include stresses of a certain degree. As an example, the parameters may be modeled based on the current economy (also known as point-in-time) which may already reflect a current recession or as an average over the business cycle (also known as through-the-cycle) which does not include any economic stresses.

With regard to the stability of the financial system, regulators provide a second safety net besides conservative assumptions for capital determination. Due to shortcomings of the models, parameter specification approaches, or potential adverse scenarios the risk models should be amended by an additional stress-testing concept similar to the one for market risk, see [13], paragraph 5:

"A bank must have in place sound stress-testing processes for use in the assessment of capital adequacy. These stress measures must be compared against the measure of expected positive exposure and considered by the bank as part of its internal capital adequacy assessment process. Stress-testing must also involve identifying possible events or future changes in economic conditions that could have unfavorable effects on a firm's credit exposures and assessment of the firm's ability to withstand such changes. Examples of scenarios that could be used are:

1. Economic or industry downturns,
2. Market-place events, or
3. Decreased liquidity conditions.

In addition to the more general tests described above, the bank must perform a credit risk stress-test to assess the effect of certain specific conditions on its IRB regulatory capital requirements[...]"

Generally, stress-testing is categorized into sensitivity tests and scenario tests, see [12]. Sensitivity tests assess the impact of made-up large movements in financial variables, while stress scenarios assess the impact of plausible but unlikely events such as historical scenarios like the U.S. stock market declines in October 1987, the Asian financial crisis of 1997, the financial market fluctuations surrounding the Russian default of 1998, and financial market developments following the September 11, 2001, terrorist attacks in the United States.

Regarding credit risk the banking practice uses two different approaches for stress-testing, see [14]. In the trading book stress-tests are similar to market risk stress-tests and adverse shocks for swap spreads, corporate bond spreads or credit default swap spreads are analyzed. In the loan book the respective variables, such as PD, LGD, or internal ratings are stressed.

There is also critique on stress-testing. As argued in [2] the fundamental problem of any stress-testing is the confusion with the basic risk modeling approach. The usual way of stress-testing takes place outside the basic risk model yielding in two sets of forecast - one from the risk model (e.g., the Value-at-Risk) and one from the stress-test. As discussed in [2] the basic problem is that probabilities are not assigned to the stress scenarios. Therefore, there is no guidance about the relevance of the stress scenarios and how to incorporate the stress forecasts into the underlying risk model.

Assigning this rationale with banking practice for loan portfolios it is plausible to use a stress-testing approach which

1. Stresses relevant variables and is therefore in accordance with common practice, <u>and</u>
2. Assigns probabilities to the stress scenarios.

The present paper makes the following contributions. Firstly, so far, there has not been a solution for stressing risk parameter forecasts for loan portfolios in a consistent manner. As an answer to this deficiency, the present paper provides a simple suggestion for stressing PDs, and therefore any derivative such as credit portfolio loss, risk measures, or regulatory capital associated with these parameters. In our approach the risk model is separated from the stress-test, as required in Basel II. For the risk model which is the basis for regulatory capital we use the "most likely" variable values. However, we additionally address the critique in [2] and show how to derive stress scenarios which are explicitly assigned probabilities. We also extend the approach to a simultaneous stress-test of PDs and correlations. This might be the basis for a stress-test of a more thorough internal credit risk model than admitted under Basel II, where additional dependency parameters may be determined by the financial institutions. Secondly, we provide an empirical application of the framework to different loan categories for which various stress scenarios are analyzed and compared. The analysis is based on data for US retail borrowers. Implications on the economic and regulatory capital are explored.

The rest of the paper is structured as follows. The next section develops a stress-testing framework based on the factor model underlying the IRB approach used by the Basel Committee on

Banking Supervision following [6] and [7]. The third section provides an empirical analysis for a through-the-cycle and a point-in-time modeling methodology and analyzes the impact of various stress levels on the economic and regulatory capital of financial institutions. The last section concludes, discusses the results and limitations of the presented research as well as extensions for future research.

2 Framework

2.1 Basic Credit Risk Model

Credit portfolio risk is often measured by a certain percentile of the distribution of future credit portfolio losses. The derivation of this percentile, also known as Value-at-Risk (VaR), is based on a variety of parameters such as the probability of default, losses given default, exposures at default or the correlation between the respective risk parameters. Given the VaR, other measures for credit portfolio risk such as Unexpected Loss (i.e., the difference between the Expected Loss and Value-at-Risk) or Expected Shortfall (i.e., the expected value of losses exceeding the Value-at-Risk) can also be derived. In this paper we will focus on two parameters namely the probabilities of default and asset correlations. A framework for other parameters is discussed in the last section.

For the default process of a borrower we use a stylized model similar to [11]. The event in which an obligor misses a payment obligation is defined as a default. The observable default event for obligor i in time period t is random and modeled using the indicator variable

$$D_{it} = \begin{cases} 1 & \text{borrower } i \text{ defaults in } t \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$(i \in N_t,\ t = 1, \cdots, T)$. N_t denotes the set of all obligors who did not default at the beginning of time period t. The time period $[1, T]$ is called the estimation period for the respective parameters.

In addition, the continuous non-observable variable R_{it} is defined, which may be interpreted as the logarithmic return on an obligor's assets. A threshold-value model is assumed for the relationship between the return on assets and the default event D_{it}. Default is equivalent to the return on assets falling below the threshold c, i.e.,

$$R_{it} < c \ \Leftrightarrow D_{it} = 1 \tag{2}$$

For the continuous non-observable variable R_{it} we assume a factor model

$$R_{it} = \omega\, F_t + \sqrt{1-\omega^2}\, U_{it} \tag{3}$$

where F_t denotes a common systematic risk factor, and U_{it} denotes an obligor-specific idiosyncratic risk factor. F_t and U_{it} are i.i.d. standard normally distributed random variables and independent of each other. [9] show that other distributions such as the logistic distribution can be chosen. Note that model (3) is assumed by the Basel Committee on Banking Supervision in its Internal-Rating-Based approach in order to calculate the regulatory capital of banks (see [5]).

Furthermore, we include time-lagged observable risk drivers into the model by assuming the linear specification for the threshold c

$$c_t = \alpha + \boldsymbol{\beta}' \boldsymbol{z}_{t-1} \tag{4}$$

where $\boldsymbol{z}_{t-1}$ denotes a vector of time-lagged systematic risk factors, like the default rate, unemployment rate or the money market rate of previous years. α and $\boldsymbol{\beta}'$ are the respective parameters.

An important feature of this specification concerns one-year forecasts because the time-lagged risk factors are known at the point in time at which the forecast is made. More details will be provided below. With this default threshold, the latent threshold variable can also be written as

$$S_{it} = R_{it} - c_t = -c_t + \omega\, F_t + \sqrt{1-\omega^2}\, U_{it} \tag{5}$$

$$= -\alpha - \boldsymbol{\beta}'\boldsymbol{z}_{t-1} + \omega\, F_t + \sqrt{1-\omega^2}\, U_{it} \tag{6}$$

where

$$S_{it} < 0 \;\Leftrightarrow D_{it} = 1 \tag{7}$$

Extensions may incorporate multiple time-varying and time-lagged obligor-specific ($\boldsymbol{x}_{it-1}$) and systematic risk factors $\boldsymbol{z}_{t-1}$ into the model

$$c_{it} = \alpha + \boldsymbol{\beta}_x'\boldsymbol{x}_{it-1} + \boldsymbol{\beta}_z'\boldsymbol{z}_{t-1} \tag{8}$$

α, $\boldsymbol{\beta}_x'$ and $\boldsymbol{\beta}_z'$ are the respective parameters, see e.g. [9] and [10]. These extensions are not considered in this contribution due to the aggregation of information over borrowers in the empirical data.

2.2 Probabilities of Default and Asset Correlations

Given the realization of the unobservable systematic factor f_t one obtains the conditional default probability

$$\pi(f_t, \boldsymbol{z}_{t-1}) = P\left(D_{it} = 1 | f_t, \boldsymbol{z}_{t-1}\right) = \Phi\left(\frac{\alpha + \boldsymbol{\beta}'\boldsymbol{z}_{t-1} - \omega f_t}{\sqrt{1-\omega^2}}\right) \tag{9}$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution function. The unconditional default probability is its mean and is given by

$$\pi_t = P(D_{it} = 1 | \boldsymbol{z}_{t-1}) = \Phi(\alpha + \boldsymbol{\beta}'\boldsymbol{z}_{t-1}) \tag{10}$$

Note that the parameter ω describes the co-movement of the conditional probabilities for given time periods. The correlation between the asset returns, i.e., the so-called asset correlation can be derived by

$$Corr(R_{it}, R_{jt}) = \omega^2 \tag{11}$$

2.3 Parameter Estimation

The described model consists of a limited set of parameters α, $\boldsymbol{\beta}$, and ω which are usually unknown and unobservable. To specify the risk model empirically these parameters can be estimated from observable data. Given the availability of historic default data, parameter estimates can be derived by maximizing the log-likelihood (with respect to F_t marginal) over all obligors and periods

$$l(\alpha, \boldsymbol{\beta}, \omega) = \sum_{t=1}^{T} \ln \int_{-\infty}^{\infty} \left\{ \prod_{i \in N_t} \pi(f_t, \boldsymbol{z}_{t-1})^{d_{it}} (1 - \pi(f_t, \boldsymbol{z}_{t-1}))^{1-d_{it}} \right\} d\Phi(f_t) \tag{12}$$

The log-likelihood contains integrals which can be solved numerically using adaptive Gauss-Hermite-quadrature ([15] or [16], pp. 5-9). It follows from the general theory of Maximum-Likelihood estimation that the estimators $\hat{\alpha}$, $\hat{\boldsymbol{\beta}}$, and $\hat{\omega}$ asymptotically exist and are consistent as well as asymptotically normally distributed (see [3], pp. 243 et seq.). Applications which specify these model types include e.g., [8], [9], [17] and [18].

2.4 Stress-Testing and the Basic Credit Risk Model

Similar to the outline in [2], this study assumes a risk model which generates a forecast distribution $g\left(l_{T+1}\right)$ for the credit portfolio loss

$$L_{T+1} = \sum_{i=1}^{n_{T+1}} LGD_{iT+1} \cdot E_{iT+1} \cdot 1_{\{S_{iT+1} \leq 0\}} \tag{13}$$

where n_{T+1} is the number of borrowers in the portfolio at the beginning of forecasting period $T+1$. The risk model can be empirically specified by substituting its unknown parameters by some empirical counterparts, e.g., the Maximum-Likelihood parameter estimates. Consider the Maximum-Likelihood parameter estimates $\hat{\alpha}$, $\hat{\boldsymbol{\beta}}$, and $\hat{\omega}$ from a historical time series. Given these estimates and the actual value of the macroeconomic variable $\boldsymbol{z}_T$, the conditional (for given f_{T+1}) and unconditional default probability can be forecast for the next year by

$$\hat{\pi}(f_{T+1}, \boldsymbol{z}_T) = \hat{P}\left(D_{iT+1} = 1 | f_{T+1}, \boldsymbol{z}_T\right) = \Phi\left(\frac{\hat{\alpha} + \hat{\boldsymbol{\beta}}' \boldsymbol{z}_T - \hat{\omega} f_{T+1}}{\sqrt{1-\hat{\omega}^2}}\right) \tag{14}$$

$$\hat{\pi}_{T+1} = \hat{P}(D_{iT+1} = 1 | \boldsymbol{z}_T) = \Phi(\hat{\alpha} + \hat{\boldsymbol{\beta}}' \boldsymbol{z}_T) \tag{15}$$

The empirical risk model then gives a distribution of the loss variable

$$\tilde{L}_{T+1} = \sum_{i=1}^{n_{T+1}} LGD_{iT+1} \cdot E_{iT+1} \cdot 1_{\{\tilde{S}_{iT+1} \leq 0\}} \tag{16}$$

with $\tilde{S}_{iT+1} = -\hat{\alpha} - \hat{\boldsymbol{\beta}}' \boldsymbol{z}_T + \hat{\omega}\, F_{T+1} + \sqrt{1-\hat{\omega}^2}\, U_{iT+1}$. For example, in the simple case where each obligor has an exposure of one monetary unit and loss given default of 100 percent the distribution of losses (more precisely, the probability function) can be calculated by

$$g(\tilde{l}_{T+1}) = \int_{-\infty}^{\infty} \begin{pmatrix} n_{T+1} \\ d_{T+1} \end{pmatrix} \hat{\pi}(f_{T+1}, \boldsymbol{z}_T)^{d_{T+1}} (1 - \hat{\pi}(f_{T+1}, \boldsymbol{z}_T))^{n_{T+1} - d_{T+1}}\, d\Phi(f_{T+1}),\ d_{T+1} = 0, \ldots, n_{T+1} \tag{17}$$

where $d_{T+1} = \sum_i^{n_{T+1}} d_{iT+1}$. Note that the loss distribution is generated in many applications by Monte-Carlo simulations. From this function a risk measure, such as the Value-at-Risk can be inferred. Since this distribution and the respective risk measures are derived as best guesses using the most likely parameter values we call them 'no stress' distributions or risk measures. Similarly, the Basel II capital can be calculated using the default probability forecasts (15) or (14) respectively.

Having derived the outcomes of the basic risk model under 'no stress' conditions, we now turn to the stress outcomes. To derive the distribution under stress we will stress all variables which are affected by uncertainty. Note that the macroeconomic variable $\boldsymbol{z}_T$ affects the borrowers with a time lag. Therefore, the current value is known in a one-year perspective and there is no need for stressing. Instead, consider the stress parameters α_{stress}, $\boldsymbol{\beta}_{stress}$, and ω_{stress} resulting in stressed conditional and unconditional default probabilities

$$\pi_{stress}(f_{T+1}, \boldsymbol{z}_T) = \Phi\left(\frac{\alpha_{stress} + \boldsymbol{\beta}'_{stress} \boldsymbol{z}_T - \omega_{stress} f_{T+1}}{\sqrt{1-\omega_{stress}^2}}\right) \tag{18}$$

$$\pi_{stress,T+1} = \Phi(\alpha_{stress} + \boldsymbol{\beta}'_{stress}\boldsymbol{z}_T) \tag{19}$$

The losses under stress can similarly be expressed as

$$L_{stress,T+1} = \sum_{i=1}^{n_{T+1}} LGD_{iT+1} \cdot E_{iT+1} \cdot 1_{\{S_{stress,iT+1} \leq 0\}} \tag{20}$$

with $S_{stress,iT+1} = -\alpha_{stress} - \boldsymbol{\beta}'_{stress}\boldsymbol{z}_T + \omega_{stress}\, F_{T+1} + \sqrt{1-\omega^2_{stress}}\, U_{iT+1}$.

In addition, an analogous solution to (17) is possible.

Note the essential difference between the outcome of the basic risk model (e.g. Expected Loss or VaR of the distribution) and the stress-test. The VaR of the risk model gives a certain percentile of the future credit portfolio loss distribution (i.e., the amount of capital a bank has to hold in order to remain solvent with a certain probability) under regular or 'most likely' conditions using the point estimates for the model parameters. The stress-test provides the Expected Loss, VaR or economic capital under unlikely, yet extreme adverse conditions. The same argument applies to the calculation of the conditional probability of default for the determination of regulatory capital (i.e., equation (9) using the asset correlation proposed by the Basel Committee on Banking Supervision, see [13]).

2.5 Simultaneous Levels of Confidence

Stressing the individual estimated parameters is the critical challenge of the presented stress-testing methodology. Stress scenarios should be realistic yet quite unlikely and pessimistic. We propose to define a level of confidence and derive the stressed values from the parameter estimates

$$\hat{\boldsymbol{\theta}} = (\hat{\alpha}\ \hat{\beta}\ \hat{\omega})' \tag{21}$$

which are asymptotically normally distributed with mean $\boldsymbol{\theta}$ and covariance matrix Σ. Various methods exist for constructing confidence intervals around the parameter estimates. Firstly, one can distribute the likelihood for parameters exceeding the stresses between the parameters with equal weights. In other words the parameter estimates can be stressed in a univariate way by taking the means and variances into account. The associated confidence intervals are also known as Bonferroni confidence intervals

$$P\left(\bigcap_j \left\{\hat{\theta}_j - t\left(n-1; 1-\frac{\gamma_j}{2}\right)\hat{s}(\hat{\theta}_j) \leq \theta_j \leq \hat{\theta}_j + \ldots\right\}\right) \geq 1-\gamma \tag{22}$$

where the number of observations of the estimation period $n = \sum_t n_t$, the standard error of the parameter estimate $\hat{\theta}_j$ equals $\hat{s}(\hat{\theta}_j)$ and the total error likelihood $\gamma = \sum_j \gamma_j$, i.e., the error likelihood is distributed equally between the parameters. Note that the error likelihood may be unequally distributed and implications studied in extensions.

Secondly, the parameters can be stressed in a multivariate way by additionally accounting for covariances between the parameter estimates by

$$P\left\{(\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}_{\boldsymbol{stress}})'\Sigma^{-1}(\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}_{\boldsymbol{stress}}) \leq \frac{(n-1)J}{(n-J)n}F\left(J; n-J; 1-\gamma\right)\right\} \geq 1-\gamma \tag{23}$$

In this approach, the stressed parameters are a function of each other. Multivariate confidence levels are more complicated to implement. This is particularly true for credit risk models with more than two parameters. As a consequence we limit our empirical analysis to Bonferroni confidence intervals.

3 Empirical Analysis

3.1 Data

The American Bankers Association kindly provided delinquency rates for selected retail loan categories for this study. Delinquency rates are defined as the ratio of delinquent loans in $ terms to total loans in $ terms for 400 representative US banks for a period ranging from 1980 to 2005. The estimation period starts in the second year of the observation period. This restriction is necessary to compare the models in section 3.2 with the models in section 3.3. The latter are based on the endogenous variables lagged by one year and require the information of the variable for the previous year. Table 1 shows the loan categories for the delinquency rates, abbreviations used in the analysis, the classification into retail exposure classes used by the Basel Committee on Banking Supervision (2006) in the IRB approach (see [13]) and the time horizon for which the information is available.

Table 1: Loan categories, abbreviations, Basel II retail exposure classes and observation periods

Loan category	**Abbreviation**	**Exposure classes**	**Periods**
Home equity loans (closed end)	HLC	Residential mortgage	1983-2005
Home equity loans (open end)	HLO		1987-2005
Bank card	BAC	Qualifying revolving	1980-2005
Non-card (revolving)	NCR		1980-2005
Automobile, direct	AMD	Other retail	1980-2005
Automobile, indirect	AMI		1980-2005
Education	EDU		1992-2005
Marine Financing	MAF		1992-2005
Mobile Home	MOH		1980-2005
Personal, unsecured	PUN		1980-2005
Property improvement	PIM		1980-2005
Recreational Vehicle	REV		1980-2005

Figure 1 shows that the delinquency rates are subject to cyclical movements. The early 90s and the beginning of this millennium can be seen as two periods with high delinquency rates and therfore high credit risk in the US economy.

Table 2 and Table 3 include descriptive statistics as well as Bravais-Pearson correlation coefficients for the time series. In line with recent quantitative impact studies by the Basel Committee, mortgage loans have generally lower delinquency rates than qualifying revolving loans.

Figure 1: Delinquency rates of different loan categories

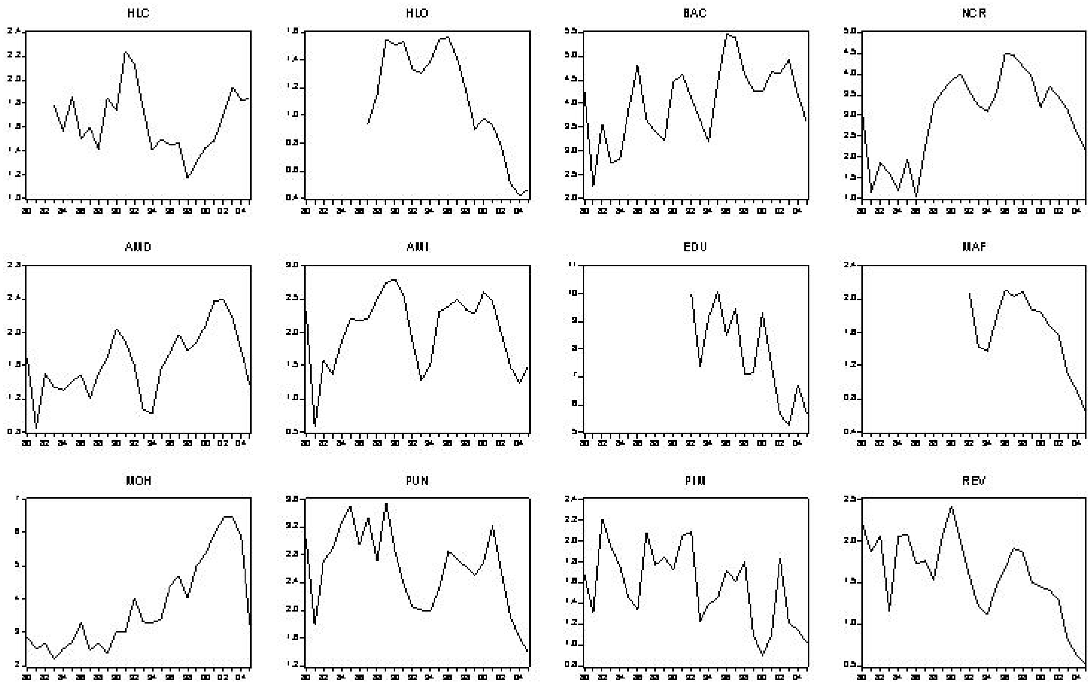

Table 2: Descriptive statistics of the delinquency rates

	Mean	**Med**	**Min**	**Max**	**STD**	**Skew**	**Kurt**	**Obs**
HLC	0.016	0.016	0.012	0.022	0.003	0.487	-0.120	22
HLO	0.011	0.012	0.004	0.016	0.004	-0.652	-0.871	18
BAC	0.040	0.042	0.023	0.055	0.008	-0.302	-0.372	25
NCR	0.030	0.032	0.011	0.045	0.010	-0.503	-0.893	25
AMD	0.016	0.016	0.009	0.024	0.004	0.073	-0.508	25
AMI	0.020	0.022	0.006	0.028	0.006	-0.713	-0.050	25
EDU	0.076	0.074	0.053	0.101	0.016	0.072	-1.217	13
MAF	0.016	0.017	0.007	0.021	0.005	-0.735	-0.371	13
MOH	0.038	0.033	0.022	0.065	0.014	0.794	-0.655	25
PUN	0.026	0.027	0.014	0.036	0.006	-0.238	-0.586	25
PIM	0.016	0.016	0.009	0.022	0.004	-0.033	-1.167	25
REV	0.016	0.016	0.005	0.024	0.005	-0.546	-0.070	25

Table 3: Bravais-Pearson correlations of the delinquency rates

	HLC	**HLO**	**BAC**	**NCR**	**AMD**	**AMI**	**EDU**	**MAF**	**MOH**	**PUN**	**PIM**	**REV**
HLC	1.000	-0.067	-0.085	-0.111	0.025	-0.239	-0.582	-0.819	-0.080	-0.245	0.249	-0.075
HLO		1.000	0.028	0.661	-0.263	0.564	0.809	0.726	-0.642	0.526	0.602	0.790
BAC			1.000	0.647	0.729	0.532	0.075	0.565	0.643	0.045	-0.103	0.000
NCR				1.000	0.594	0.583	0.419	0.927	0.445	-0.012	0.039	0.057
AMD					1.000	0.556	-0.259	0.269	0.759	0.164	-0.097	-0.011
AMI						1.000	0.519	0.846	0.055	0.635	0.192	0.501
EDU							1.000	0.582	-0.428	0.402	0.090	0.573
MAF								1.000	-0.017	0.865	0.497	0.981
MOH									1.000	-0.270	-0.455	-0.466
PUN										1.000	0.384	0.673
PIM											1.000	0.533
REV												1.000

3.2 Risk segments defined by product category (through-the-cycle)

In a first step, a model without any time-varying risk drivers is estimated. Since no time-varying information is included, the delinquency rates are modeled by the average state of the business cycle. Such a model is often called a through-the-cycle model. Table 4 shows the resulting parameter estimates as well as the derived probabilities of default (PD) and asset correlations (AC):

Table 4: Estimates and standard errors for the parameters α and ω; probabilities of default are calculated based on formula (10); asset correlations are calculated based on formula (11); through-the-cycle modeling methodology

Parameter	**PD**	$\hat{\alpha}$	$\hat{\alpha}$ **SE**	**AC**	$\hat{\omega}$	$\hat{\omega}$ **SE**
HLC	0.016	-2.133	0.013	0.004	0.062	0.010
HLO	0.011	-2.275	0.037	0.023	0.152	0.025
BAC	0.040	-1.746	0.019	0.009	0.096	0.014
NCR	0.030	-1.882	0.036	0.031	0.176	0.024
AMD	0.016	-2.133	0.020	0.010	0.100	0.014
AMI	0.020	-2.050	0.028	0.018	0.135	0.019
EDU	0.076	-1.433	0.030	0.011	0.107	0.021
MAF	0.016	-2.150	0.037	0.017	0.130	0.025
MOH	0.038	-1.776	0.031	0.023	0.151	0.021
PUN	0.026	-1.948	0.021	0.010	0.101	0.014
PIM	0.016	-2.154	0.020	0.009	0.097	0.014
REV	0.016	-2.151	0.029	0.019	0.139	0.019

Given the nature of most retail portfolios, we assume an infinitely granular portfolio, a portfolio exposure of one and a loss given default of one. As a result, the unconditional probability of default can be interpreted as the Expected Loss and the conditional probability of default as the Value-at-Risk.

In accordance with the Basel Committee on Banking Supervision (see [13]) the 99.9th percentile is used as the realization of the systematic random factor. The Expected Loss will then be compared to the Value-at-Risk based on this percentile for various stress levels. As mentioned in section 2, the presented results are based on Bonferroni confidence intervals with a total error likelihood (stress) of 0.1, 0.05, 0.01 and 0.001.

Table 5 shows the resulting Expected Loss, Value-at-Risk and Basel II (BII) Value-at-Risk.

Table 5: Expected loss, Value-at-Risk and Basel II Value-at-Risk based on univariate confidence intervals; through-the-cycle modeling methodology

The stressed probability of default (i.e., the Expected Loss given an infinitely granular portfolio with a total exposure and loss given default of one) is calculated using formula (19). The stressed conditional probability of default (i.e., the Value-at-Risk given an infinitely granular portfolio with a total exposure and loss given default of one) is calculated using formula (18). The Basel II Value-at-Risk is calculated by replacing the estimated stressed parameter $(\hat{\omega}_{stress})^2$ by the asset correlation proposed by the Basel Committee on Banking Supervision.

	HLC	**HLO**	**BAC**	**NCR**	**AMD**	**AMI**	**EDU**	**MAF**	**MOH**	**PUN**	**PIM**	**REV**
Expected loss												
no stress	0.017	0.011	0.040	0.030	0.016	0.020	0.076	0.016	0.038	0.026	0.016	0.016
stress = 0.10	0.018	0.014	0.044	0.035	0.018	0.023	0.086	0.019	0.043	0.028	0.017	0.018
stress = 0.05	0.018	0.015	0.045	0.036	0.019	0.024	0.088	0.020	0.044	0.029	0.018	0.019
stress = 0.01	0.018	0.016	0.046	0.038	0.019	0.025	0.092	0.022	0.046	0.030	0.018	0.020
stress = 0.001	0.019	0.017	0.048	0.041	0.020	0.026	0.098	0.024	0.049	0.031	0.019	0.021
Value-at-Risk												
no stress	0.026	0.034	0.073	0.087	0.033	0.050	0.134	0.039	0.093	0.050	0.031	0.041
stress = 0.10	0.032	0.055	0.091	0.128	0.044	0.070	0.182	0.064	0.129	0.065	0.041	0.059
stress = 0.05	0.033	0.059	0.095	0.135	0.046	0.073	0.192	0.069	0.135	0.068	0.043	0.062
stress = 0.01	0.035	0.070	0.102	0.152	0.050	0.082	0.215	0.082	0.150	0.073	0.047	0.070
stress = 0.001	0.038	0.087	0.112	0.177	0.056	0.094	0.251	0.106	0.171	0.082	0.052	0.081
BII Value-at-Risk												
no stress	0.155	0.121	0.125	0.099	0.114	0.124	0.201	0.112	0.153	0.135	0.112	0.112
stress = 0.10	0.162	0.139	0.133	0.113	0.123	0.136	0.220	0.129	0.169	0.144	0.120	0.124
stress = 0.05	0.164	0.142	0.135	0.115	0.124	0.138	0.224	0.132	0.172	0.146	0.122	0.126
stress = 0.01	0.166	0.149	0.138	0.120	0.127	0.143	0.232	0.140	0.177	0.149	0.125	0.131
stress = 0.001	0.170	0.159	0.142	0.127	0.132	0.149	0.244	0.151	0.185	0.154	0.129	0.137

In the Internal-Ratings-Based approach of the Basel Committee, the regulatory capital can be calculated by multiplying the respective Value-at-Risk with the loss given default. The same is true for the economic capital under the assumption of independence between the default events and losses given default (for a critique compare [18]). For example, given an AMI loan, a loss given default of 50 percent and an error likelihood of 0.001 the economic Value-at-Risk would be $0.094 \times 0.5 = 4.7$ percent and the Basel II Value-at-Risk $0.149 \times 0.5 = 7.45$ percent of the loan exposure. It can be seen in Table 5 that both values depend on the level of confidence. The regulatory capital will be lower for the risk segments NCR and EDU and therefore insufficient to cover the economic capital if it is based on the stressed parameter estimates with an error likelihood of 0.001. This is due to the fact that the asset correlations proposed by the Basel Committee are lower than the stressed empirical estimates. The problem increases if the regulatory capital is based on unstressed parameters. However, this conclusion does not take other stresses such as stressed losses given default into account. Table 6 shows that the estimated and stressed asset correlation exceeds the assumed values of the Basel Committee for these risk segments.

Table 6: Empirical and Basel II asset correlations; through-the-cycle modeling methodology

	HLC	**HLO**	**BAC**	**NCR**	**AMD**	**AMI**	**EDU**	**MAF**	**MOH**	**PUN**	**PIM**	**REV**
no stress	0.004	0.023	0.009	0.031	0.010	0.018	0.012	0.017	0.023	0.010	0.010	0.019
stress = 0.10	0.007	0.042	0.015	0.051	0.017	0.030	0.023	0.034	0.038	0.017	0.016	0.032
stress = 0.05	0.007	0.046	0.017	0.055	0.018	0.032	0.026	0.038	0.041	0.018	0.017	0.034
stress = 0.01	0.009	0.054	0.019	0.063	0.021	0.037	0.032	0.047	0.047	0.021	0.020	0.040
stress = 0.001	0.010	0.067	0.023	0.075	0.025	0.044	0.042	0.062	0.056	0.025	0.023	0.047
Basel II	0.150	0.150	0.040	0.040	0.103	0.094	0.039	0.105	0.065	0.083	0.105	0.105

3.3 Risk segments defined by product category and economic information (point-in-time rating)

In a second step, the delinquency rates will be modeled by the delinquency rates in the previous year, i.e., an AR (1) process. The time-lagged endogenous variable is a good proxy for the current state of the business cycle. The variables were significant at the 1 percent level in eight models, significant at the 5 percent level in two models (BAC and AMI) and significant at the 10 percent level in two models (EDU and PIM). In extensions, macroeconomic as well as idiosyncratic risk drivers may be included. Such a model is often called a point-in-time model. Table 7 shows the resulting parameter estimates as well as the asset correlations. Note that contrary to Table 4, the probabilities of default are not stated since in a point-in-time model these estimates are generally different in every year.

Table 7: Estimates and standard errors for the parameters α, β and ω; asset correlations are calculated based on formula (11); point-in-time modeling methodology

Parameter	$\hat{\alpha}$	$\hat{\alpha}$ **SE**	$\hat{\beta}$	$\hat{\beta}$ **SE**	**AC**	$\hat{\omega}$	$\hat{\omega}$ **SE**
HLC	-2.362	0.070	13.873	4.233	0.003	0.050	0.008
HLO	-2.758	0.055	39.604	4.515	0.004	0.066	0.011
BAC	-1.975	0.090	5.591	2.169	0.007	0.086	0.012
NCR	-2.292	0.074	13.083	2.337	0.014	0.118	0.017
AMD	-2.392	0.070	15.351	4.092	0.006	0.080	0.011
AMI	-2.274	0.095	10.818	4.464	0.015	0.121	0.017
EDU	-1.699	0.138	3.342	1.710	0.009	0.094	0.018
MAF	-2.642	0.098	28.516	5.669	0.006	0.075	0.015
MOH	-2.148	0.052	9.470	1.290	0.007	0.086	0.012
PUN	-2.247	0.085	11.218	3.149	0.007	0.082	0.012
PIM	-2.318	0.083	10.208	5.132	0.008	0.090	0.013
REV	-2.536	0.079	22.946	4.651	0.010	0.099	0.014

A comparison of Table 7 and Table 4 shows that the standard error of the intercept is larger for the point-in-time model than for the through-the-cycle model. In addition, consistent with previous studies (see [17], [18] and [9]) the estimated asset correlation and standard errors are smaller for the point-in-time model than for the through-the-cycle model.

The high standard errors for the intercept lead to higher stressed probabilities of default (or expected losses) and as a consequence to a higher Value-at-Risk in economic as well as regulatory terms. Table 8 shows the resulting Expected Loss, Value-at-Risk and Basel II (BII) Value-at-Risk. In order to compare the results to the through-the-cycle model in the previous section, the average delinquency rate was used as the realization of the time-lagged endogenous variable.

Table 8: Expected loss, Value-at-Risk and Basel II Value-at-Risk based on univariate confidence intervals; point-in-time modeling methodology

The stressed probability of default (i.e., the Expected Loss given a total exposure and loss given default of one) is calculated using formula (19). The stressed conditional probability of default (i.e., the Value-at-Risk given a total exposure and loss given default of one) is calculated using formula (18). The Basel II Value-at-Risk is calculated by replacing the estimated stressed parameter $(\hat{\omega}_{stress})^2$ by the asset correlation proposed by the Basel Committee.

	HLC	**HLO**	**BAC**	**NCR**	**AMD**	**AMI**	**EDU**	**MAF**	**MOH**	**PUN**	**PIM**	**REV**
Expected loss												
no stress	0.016	0.011	0.040	0.029	0.016	0.020	0.076	0.015	0.037	0.026	0.016	0.015
stress = 0.10	0.035	0.020	0.089	0.058	0.034	0.051	0.219	0.045	0.059	0.058	0.037	0.035
stress = 0.05	0.038	0.022	0.098	0.064	0.037	0.058	0.250	0.052	0.063	0.064	0.042	0.039
stress = 0.01	0.047	0.027	0.121	0.077	0.046	0.074	0.332	0.072	0.072	0.080	0.053	0.049
stress = 0.001	0.062	0.035	0.157	0.098	0.059	0.101	0.469	0.114	0.085	0.106	0.071	0.065
Value-at-Risk												
no stress	0.024	0.018	0.068	0.061	0.029	0.045	0.125	0.027	0.063	0.044	0.030	0.031
stress = 0.10	0.054	0.038	0.158	0.135	0.066	0.124	0.362	0.087	0.111	0.107	0.077	0.078
stress = 0.05	0.060	0.042	0.175	0.149	0.073	0.140	0.410	0.102	0.119	0.119	0.087	0.087
stress = 0.01	0.075	0.053	0.215	0.182	0.091	0.180	0.525	0.143	0.139	0.149	0.111	0.111
stress = 0.001	0.100	0.071	0.278	0.234	0.119	0.244	0.693	0.226	0.169	0.197	0.151	0.149
BII Value-at-Risk												
no stress	0.155	0.116	0.124	0.096	0.114	0.123	0.201	0.111	0.151	0.134	0.112	0.111
stress = 0.10	0.251	0.178	0.228	0.165	0.190	0.236	0.433	0.232	0.213	0.238	0.205	0.197
stress = 0.05	0.267	0.188	0.246	0.177	0.202	0.256	0.475	0.256	0.223	0.256	0.221	0.211
stress = 0.01	0.304	0.213	0.287	0.205	0.232	0.301	0.571	0.317	0.245	0.296	0.258	0.245
stress = 0.001	0.357	0.251	0.346	0.246	0.274	0.366	0.707	0.417	0.276	0.354	0.312	0.295

Under the assumption of independence between the default events and losses given default, the regulatory capital based on the stressed parameters will be sufficient to cover the economic loss for all analyzed stresses. Note that this observation does not hold for most risk segments if the regulatory capital is based on unstressed parameters.

Table 9: Empirical and Basel II asset correlations; point-in-time modeling methodology

	HLC	**HLO**	**BAC**	**NCR**	**AMD**	**AMI**	**EDU**	**MAF**	**MOH**	**PUN**	**PIM**	**REV**
no stress	0.003	0.004	0.007	0.014	0.006	0.015	0.009	0.006	0.007	0.007	0.008	0.010
stress = 0.10	0.005	0.009	0.013	0.024	0.011	0.026	0.019	0.012	0.013	0.012	0.014	0.017
stress = 0.05	0.005	0.009	0.014	0.026	0.012	0.027	0.021	0.014	0.014	0.013	0.015	0.018
stress = 0.01	0.006	0.011	0.016	0.030	0.014	0.031	0.026	0.017	0.016	0.015	0.018	0.021
stress = 0.001	0.007	0.014	0.019	0.035	0.016	0.037	0.034	0.022	0.019	0.017	0.021	0.025
Basel II	0.150	0.150	0.040	0.040	0.104	0.095	0.039	0.106	0.066	0.083	0.105	0.106

4 Discussion

Financial institutions are faced with the challenge to forecast future credit portfolio losses. It is common practice to focus on a limited set of parameters, such as the probability of default, asset correlation, loss given default or exposure at default. With regard to the stability of the financial system, these models have to be approved by regulators who have an interest in a conservative assessment of the credit portfolio risk and require the stress-test of risk estimates. The study was motivated by the limited guidance available in the credit risk community which encompasses practitioners, regulators and academics.

The present paper has developed a framework to stress the smallest building block, the sensitivities of risk drivers and therefore any derivative such as risk parameter or credit portfolio loss. A univariate and a multivariate approach for stressing the credit risk parameters were shown in which estimation uncertainties as well as the asset correlations were taken into account. The framework can be applied to any risk segment such as corporate or retail loans, domestic or overseas loans, secured or unsecured loans as well as senior or junior loans. It is basically robust to different information levels and models. The presence of a statistical model is not required as long as the the default histories for the risk segment under consideration are available.

However, the study focused on the probabilities of default and asset correlations of loan portfolios. Other credit risk parameters such as exposures, losses given default or correlations between these random variables exist. The authors intend to extend the framework for other risk parameters in their future research. In addition, the framework may be extended to incorporate obligor-specific information which has been done in a different context in previous research but was not included into the analysis due to data constraints.

The empirical analysis was enabled by US retail loan data provided by the American Bankers Association. The application of the theoretical framework was demonstrated, Stress scenarios for different loan categories were analyzed and the implications on economic as well as regulatory capital were shown. Risk segments where the regulatory capital is below the economic capital were identified for the two distinct modeling methodologies, through-the-cycle and point-in-time. Both methodologies are recognized by the Basel Committee on Banking Supervision.

Similar empirical studies may be conducted for other risk segments such as corporate loans or other countries. As a consequence, the authors would like to like to call upon the credit community to share their experience, feedback and results.

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